~~Indian~~ ~~80%~~ ~~70%~~ Tutorial - I Physics (PH-101)

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Q.1: How is the concept of relativity applied to the mass of bod

Q.2: How is the negative results of the experiment interpreted

Q.3: What is theory of relativity and what is its need?

Q.4: Derive transformation formulas for relativistic momentum and energy and hence Prove that the quantity $(P^2 - \frac{E^2}{c^2})$ is an invariant.

Q.5 An e has a charge 1.6×10^{-19}C when it is at rest. What will be its charge if it moves with a speed of 0.8c ?

Q.6 Calculate the speed of a Particle whose total energy is exactly twice its rest energy.

Q.7: What is the error involved in calculating the K.E of a Particle according to (i) classical Physics (ii) Relativistic Physics when it is moving at $u = 0.5c$?

Q.8: Half life of a Particle at rest is 17.8 nanosecond. What wi be the half-life when its speed is 0.8 c ?

Q.9: The Length of a rod is found to be half of its length when at rest. What is the speed of the rod relative to be observe

Q.10: Calculate the velocity at which electron mass is $\sqrt{3}$ tim the rest mass?

can elongate or reduce the size of a rod without touching it. This shows that length is relative.

Similarly we can shows that motion and time is relative.

Thus it is obvious that the motion of a body has no meaning unless it is described w.r.t some some well-defin co-ordinate system, known as frame of reference with resp to which the velocity of the body is measured.

1.2. Frames of Reference:

We should know, what is meant by motion.

"The change of Position of a body relative to something els constitute motion."

For example: When we stand on a railway Platform and a train Passes by the Position of the train w.r.t the Platform changes and therefore we say the train is in motion. The changes in the Position of the train are being referred to w.r.t the Platform. The Platform is said to constitute a fra of reference. Hence anything w.r.t which the motion is descri is called a frame of reference.

"So, a frame of reference is a part of the description of the motion."

There are two types of frame of reference

(i) Interial or unaccelerated frame of reference:-

Simply a frame of reference is the cartesian co-ordinat system in which the the Position of a moving Particle P at any instant is expressed in terms of the co-ordinates (x,y,z) or by the Position vector

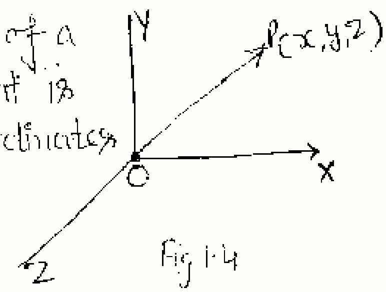


Fig 1.4

$$\vec{OP} = \vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$$

from the origin (0,0,0) as shown in fig 1.5

The velocity of the particle is then

$$\vec{v} = \frac{d\vec{r}}{dt} = \hat{i}\frac{dx}{dt} + \hat{j}\frac{dy}{dt} + \hat{k}\frac{dz}{dt}$$

and acceleration is

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} = \hat{i}\frac{d^2x}{dt^2} + \hat{j}\frac{d^2y}{dt^2} + \hat{k}\frac{d^2z}{dt^2}$$

The frames of reference in which Newton's law holds are called the inertial frames.

According to Newton's law,

The frame in which a body at rest or moving with uniform velocity and not under the influence of any force, remain at rest or moving with the same uniform velocity.

Is the earth an inertial frame ?

Strictly, we can say earth is not an inertial frame of reference. The earth is rotating not only about its own axes but also orbiting around the sun and centripetal acceleration are present.

If the body has no external force acts, its acceleration remain zero. Mathematically if F=0 in an external frame then

$$\frac{d^2r}{dt^2} = 0 \text{ or } \frac{d^2x}{dt^2} = 0, \frac{d^2y}{dt^2} = 0 \text{ \& } \frac{d^2z}{dt^2} = 0$$

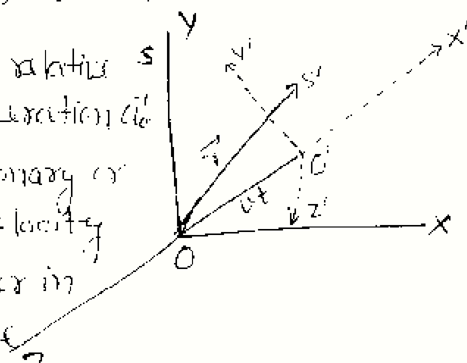
Example for inertial frames —

The interior of a vehicle, such as train, car etc. moving smoothly and with a constant velocity on its surface.

"The laws of Physics will be same for all observers in this frame of reference or the laws of mechanics are the same in all inertial frames of reference"

(ii) Non-inertial or accelerated frames of reference:-

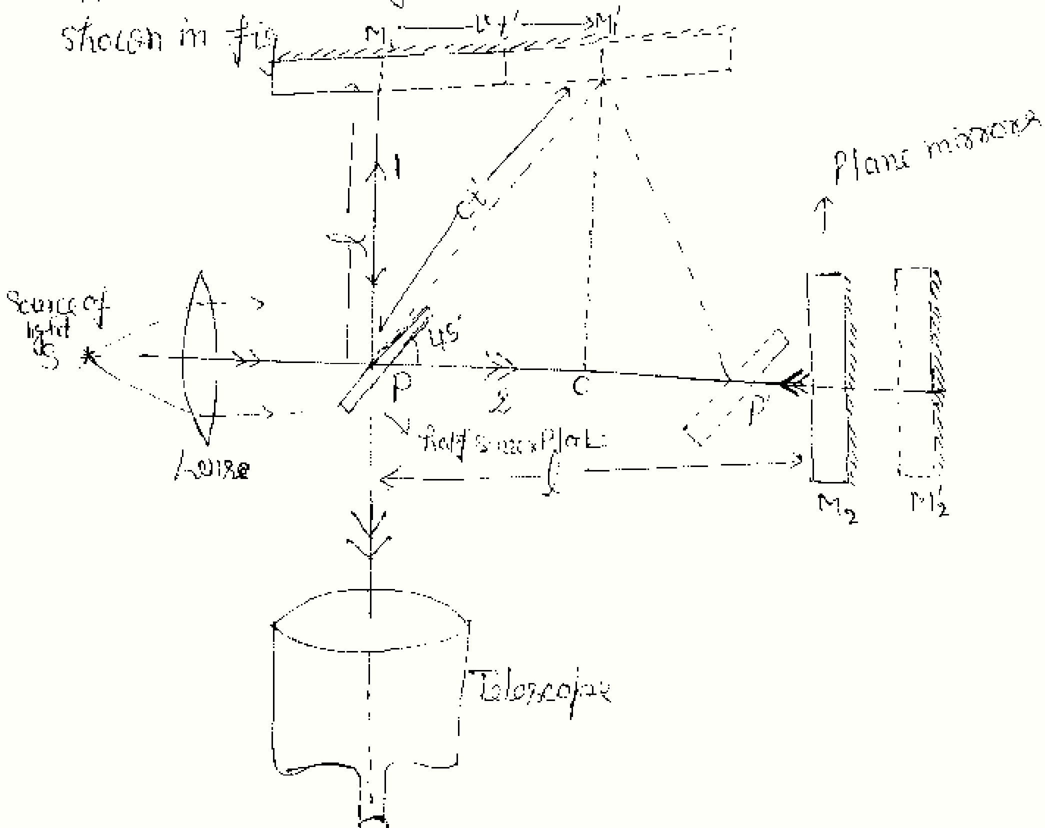
The frames of reference in which Newton's laws are not valid are called the non-inertial frames. All the rotatory and accelerated frames are non-inertial.



Let a frame S' is moving relative to a frame S with an acceleration $\vec{a}_0$. A Particle which is stationary or moving with a constant velocity with respect to an observer in frame S will appear to be moving with an acceleration $-\vec{a}_0$ to an observer in fra[me S'.] So the frame of observer S' will experience a force $-m\vec{a}_0$ [on] the Particle. or $F = -m\vec{a}_0$. This is called the fictitious [force.] This force is not real. The direction of fictitious is o[pposite] to the direction of relative acceleration of the frame of reference. Hence the frame S' is non-inertial. Since the law does not hold in it

1.3 MICHELSON-MORLEY EXPERIMENT:-

Michelson-Morley Performed a very significant and extremely sensitive experiment in 1887 using Michelson interferometer. This experiment is used for measuring the absolute velocity of the earth with respect to stationary ether. it is a greater experiment in Physics and one of the main experiment pillar of special theory of relativity. The essential features of this apparatus, universally known as Michelson interferometer are shown in fig.



M_1 and M_2 are used two Plane mirrors which is highly silvered on their front surfaces to avoid multiple internal reflection. A beam of light from the source S is incident upon a half silvered glass Plate P placed at 45° to the beam. It splits into two beams 1 and 2. These beams travel right angles (90°) to each other. They incident normally on mirror M_1 and M_2 are reflected back to P. The two beams returned to P are directed toward a telescope T and interference takes place and fringes are observed in the telescope.

Let the mirrors M_1 and M_2 at the same distance l from the glass plate P. The two beams would take the same time to return to P. Let us assume the earth is moving in space through the aether with a velocity (v) together the apparatus moving velocity v.

Suppose c is the velocity of light through the ether. The beam is moving towards M_2 has a velocity $(c-v)$ relative to the apparatus on the out going journey PM_2 and $(c+v)$ on the return journey. If t_2 is the total time by this beam to go from P to M_2 and back, then

$$t_2 = \frac{l}{c-v} + \frac{l}{c+v} = \frac{2lc}{c^2-v^2}$$

$$t_2 = \frac{2lc}{c^2\left(1-\frac{v^2}{c^2}\right)} = \frac{2l}{c}\cdot\frac{1}{\left(1-\frac{v^2}{c^2}\right)} \quad \text{--- (1)}$$

If t' be the time taken by the beam in going from the point P to M_1, then the distance travelled by ct'. The mirror M_1 shifted M_1' after covering a horizontal distance vt'.

therefore $\triangle PM_1M_1'$

$$(PM_1')^2 = (PM_1)^2 + (M_1M_1')^2$$

$$(ct')^2 = l^2 + (vt')^2$$

$$(c^2-v^2)\,t'^2 = l^2 \quad \text{or} \quad t'^2 = \frac{l^2}{c^2-v^2}$$

$$\text{or } t' = \frac{l}{c}\cdot\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Hence the total time taken by the beam in travelling from P to M_1' and then from M_1' to P'.

$$t_1 = 2t' = \frac{2l}{c} \cdot \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{---} (2)$$

therefore the time difference Δt

$$\Delta t = t_2 - t_1 = \frac{2l}{c} \cdot \frac{1}{\left(1-\frac{v^2}{c^2}\right)} - \frac{2l}{c} \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Delta t = \frac{2l}{c}\left[\left(1-\frac{v^2}{c^2}\right)^{-1} - \left(1-\frac{v^2}{c^2}\right)^{-1/2}\right]$$

By using binomial theorem

$$(1+x)^n = \left(1 + nx + \frac{n(n-1)}{\underline{2}} + \cdots\right) \text{ neglected higher term}$$

we get

$$\Delta t = \frac{2l}{c}\left[\left(1+\frac{v^2}{c^2}+\cdots\right) - \left(1+\frac{1}{2}\frac{v^2}{c^2}+\cdots\right)\right]$$

$$\text{"} = \frac{2l}{c}\left[\cancel{1}+\frac{v^2}{c^2} - \cancel{1} - \frac{1}{2}\frac{v^2}{c^2}\right]$$

$$\Delta t = \frac{2l}{c} \cdot \frac{1}{2}\frac{v^2}{c^2}$$

$$\Delta t = \frac{lv^2}{c^3} \quad \text{-----} (3)$$

$\therefore$ The corresponding Path difference is

$$\delta = c\Delta t = \frac{lv^2}{c^2} \quad \text{---} (4)$$

We know that if the Path difference between the two interfering rays changes by λ. if n is the number of fringes that shift when interferometer is suddenly brought to rest.

From the equation (4), we becomes,

$$n = \frac{\delta}{\lambda} = \frac{l v^2}{c^2 \lambda} \quad \text{---(5)}$$

If the apparatus is turned through 90°, so the path PM_1 become longer than the path PM_2 by an amount in $\frac{l v^2}{c^2}$. Now the role of paths 1 & 2 are interchanged, In this case, the time t_2' and t_1' required to travel the paths $PM_1'P'$ then

$$t_2' = \frac{2l}{c} \cdot \frac{1}{\left(1-\frac{v^2}{c^2}\right)^{1/2}} \quad \text{---(6)}$$

and

$$t_1 = \frac{2l}{c} \cdot \frac{1}{\left(1-\frac{v^2}{c^2}\right)} \quad \text{---(7)}$$

Now the time difference

$$\Delta t' = t_2' - t_1' = \frac{2l}{c}\left[\left(1-\frac{v^2}{c^2}\right)^{-1/2} - \left(1-\frac{v^2}{c^2}\right)^{-1}\right]$$

By using Binomial theorem

$$= \frac{2l}{c}\left[1 + \frac{1}{2}\frac{v^2}{c^2} - 1 - \frac{v^2}{c^2}\right]$$

$$\Delta t' = \frac{2l}{c}\left(-\frac{1}{2}\frac{v^2}{c^2}\right) = -\frac{l v^2}{c^3} \quad \text{---(8)}$$

$$\therefore \Delta T_{total} = \Delta t - \Delta t' = \frac{l v^2}{c^3} - \left(-\frac{l v^2}{c^3}\right)$$

$$\boxed{\Delta T_{total} = \frac{2 l v^2}{c^3}} \quad \text{----(9)}$$

The path difference introduced b/w beam 1 & 2 will be,

$$\delta = c \cdot \Delta T_{total} = c \cdot \frac{2 l v^2}{c^3} = \frac{2 l v^2}{c^2} \quad \text{---(10)}$$

The number of fringes

and $$N = \frac{\delta}{\lambda} = \frac{2 l v^2}{c^2 \lambda} \quad \text{-----(11)}$$

where λ is the wavelength of light used.

A Michelson and Morley had taken $\lambda = l = 11$ metre by reflecting the light back and forth several times $\lambda = 6000 Å$ (for visible light) $v = 3 \times 10^4$ m/sec it is orbital velocity of earth. The number of fringes

$$N = \frac{2 \times 11 \times (3 \times 10^4)^2}{(3 \times 10^8)^2 \times 6 \times 10^{-7}} = 0.37 \simeq .4$$

A fringe shift of this amount is easily detected with the apparatus. It should be possible to measure the fringe shift and from it determine the velocity of the earth relative to the ether.

Note: The measurements were made during day and night in various seasons throughout the year. The fringes are observed by rotating the apparatus continuously. The results of the experiment are always proved to be negative. It is also showed that motion relative to the ether cannot be detected and the velocity of light is independent of the motion of the light source. Since its motion with respect to ether is different at different season of the year.

* Ether is an all pervading, infinitely elastic, massless medium relative to which the measurement are taken.

Explanation of the negative results:-

(1) Ether - Drag Hypothesis:- Michelson tried to explain this result on the hypothesis that the earth dragged the ether along with it, so that there was no relative motion b/w the earth and ether. Michelson explanation was, therefore not tenable

(2) Lorentz and Fitzgerald : According to them of negative result of ~~M-M experiment~~ a moving body is altered due to its motion through stationary ether. They showed that the length of the Path in the direction of ether flows will be shortened by $l\sqrt{1-u^2/c^2}$ instead of l. and replacing l by $l\sqrt{1-\frac{u^2}{c^2}}$ in eq(2), we get

$$t_2 = \frac{2l\sqrt{1-\frac{u^2}{c^2}}}{c\left(1-\frac{u^2}{c^2}\right)} = \frac{2l}{c\sqrt{1-\frac{u^2}{c^2}}}$$

when $t_1 = \frac{2l}{c\sqrt{1-\frac{u^2}{c^2}}}$ Hence $t_2 = t_1$

(3) Constancy of speed of light hypothesis :-

It was Proposed that light travels with a constant velocity not w.r.t the stationary ether but w.r.t the source. Thus, the light from a moving source has a velocity which is the vector sum of its natural velocity and the velocity of source. This ~~is~~ explains the -ve result

Einstein's interpretation of Mi-Mo Experiment :-

Einstein Proposed that the basic concepts of Mi-M. experiment were wrong. His argued that the motion thro-ugh stationary ether is a meaningless Only motion relative to material bodies has Physical significance while discussing any motion the frame of reference must be specified, which may be road, the earth's motion of the surface, the sun the centre of our galaxy etc.

Postulates of special Theory of Relativity:-

There are two Postulates of the special theory of relativity proposed by Einstein:

(1) The laws of Physics are the same in all inertial frames of reference moving with a constant velocity w.r.t one another. In Einstein's our words... "the same laws of electrodynamics and optics will be valid for all frames of reference ~~for all frames of reference~~ for which the equations of mechanics hold good"

(2) The speed of light in free space has the same value in all inertial frames of reference. This speed is 2.9979×10^{8} m/sec. "This Postulate is directly followed from the result of Michelson-Morley experiment.

Galilean Transformations:-

The equations relating the co-ordinates of a Particle in two inertial frames are called the Galilean Transformations.

Galilean Transformations are used to transform the Co-ordinates of Position (x,y,z) and time(t) from one inertial frame to another.

Let us consider two frames of reference S and S' are shown in fig.

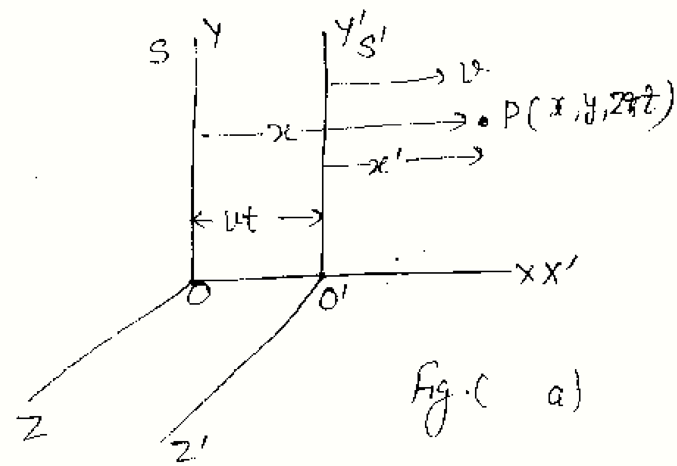


Fig.(a)

Let the velocity of frame S' relative to S be u at any particular instant.

Let an event occur at P. The co-ordinates of P w.r.t S are x, y, z and t And w.r.t to S' are x', y', z' & t'. We have to find out the relationship between these two co-ordinates. Let the axes X and X' be parallel to u and Y Z-Z axes be parallel to Y' & Z' respectively. The time also measured from the instant at which the origins O and O' coincide Fig (a) shows that

$x' = x - ut$, $y' = y$, $z' = z$

and also $t' = t$ —— (1)

These equation are known as Galilean transformation eq^n

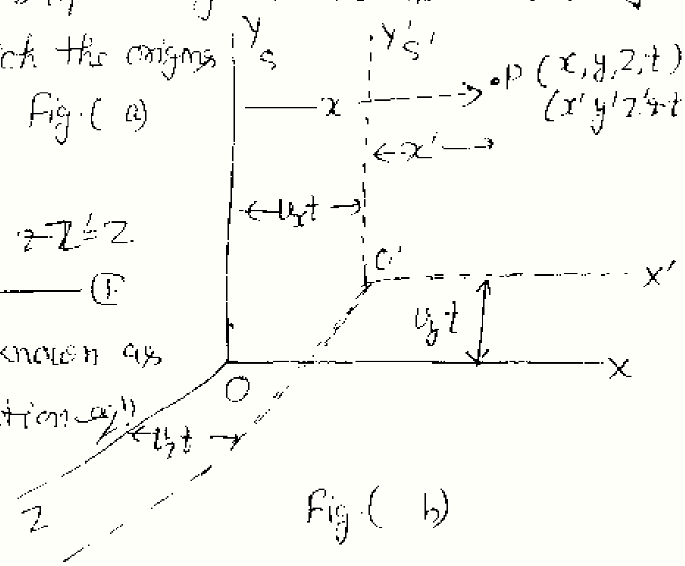


Fig (b)

shown the above Fig (b). The two following situation is

(i) When frame S' is moving along a straight line relative to S in any direction:— The frame S' is moving relative to S with velocity u. s.t $\vec{u} = u_x\hat{i} + u_y\hat{j} + u_z\hat{k}$

where u_x, u_y & u_z are component of velocities in x, y, z direc

Let the origin of O & O' coincide at any instant, let an even happen at P. Here the co-ordinates of P w.r.t S and S' are x, y, z, t and x', y', z', t' respectively.

(ii) when S' is separated from S by distance $v_x t, v_y t, v_z$ along three axes respectively, we have

$$x' = x - v_x t,\ y' = y - v_y t,\ z' = z - v_z t \text{ and } t' = t \quad \text{--- (2)}$$

For Galilean transformation of the velocities of the Part let a Particle be observed at P.

On differentiating eqn (2), we get

$$dx' = dx - v_x dt,\ dy' = dy - v_y dt,\ dz' = dz - v_z dt \text{ and } dt' = dt \quad \text{--- (3)}$$

here v_x, v_y, v_z are Constant.

From eqn (3), we get

$$\frac{dx'}{dt} = \frac{dx}{dt} - v_x,\ \frac{dy'}{dt} = \frac{dy}{dt} - v_y,\ \frac{dz'}{dt} = \frac{dz}{dt} - v_z \quad \text{--- (4)}$$

$$\therefore u'_x = u_x - v_x,\ u'_y = u_y - v_y,\ u'_z = u_z - v_z \quad \text{--- (5)}$$

here u_x, u_y, u_z are the velocities of the Particle in frame S
and u'_x, u'_y, u'_z " " " " " " " " S'

if $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along x, y, z axes, then

$$u'_x\hat{i} + u'_y\hat{j} + u'_z\hat{k} = u_x\hat{i} + u_y\hat{j} + u_z\hat{k} - v_x\hat{i} - v_y\hat{j} - v_z\hat{k}$$

$$\text{or } \boxed{u' = u - v} \quad \text{--- (6)}$$

This is a Galilean transformation of the velocity of the Par

For accelerative transformation, as v is a Constant velo

$$\boxed{\Delta u' = \Delta u}$$

Hence, the change in velocity observed from S' is equa, to the velocity change observed from S.

Again $a_x' = \frac{\Delta u'}{\Delta t'} = \frac{\Delta u}{\Delta t} = a_x$ --- (7)

Similarly $a_y' = a_y$ and $a_z' = a_z$ (8)

Hence acceleration of the Particle in frame S' is the same as that in frame S.

In other words

Inertial frames move with Constant velocities relative to each other.

Lorentz Transformation of ~~Ship~~ Space and Time—

On the basis of Einstein Postulates of special theory of relativity Lorentz derived algebrically the transformation that has replaced Galilean transformation and also agrees with the Michelson-Morley Experiment.

Let us consider a system of two inertial frames of reference S and S'. Let S' is moving with velocity v w.r.t to S in Positive x-direction. let two observers situated at O and O' (origins w.r.t S & S') are observing any event P. Let us assume that the x-axis of two systems coincide permanently. And the velocity v is Parallel to axis. The event P is determined by the co-ordinates x, y, z, t for an observer O on the frame S while the same event is determined by co-ordinate x', y', z', t' for an observer O' on the moving frame S'. Let the time be counted from the instant when the origins O & O' momentarily coincide.

when the event is observed by S, we have

$$\text{Velocity of light} = \frac{\text{Distance}}{\text{Time}}$$

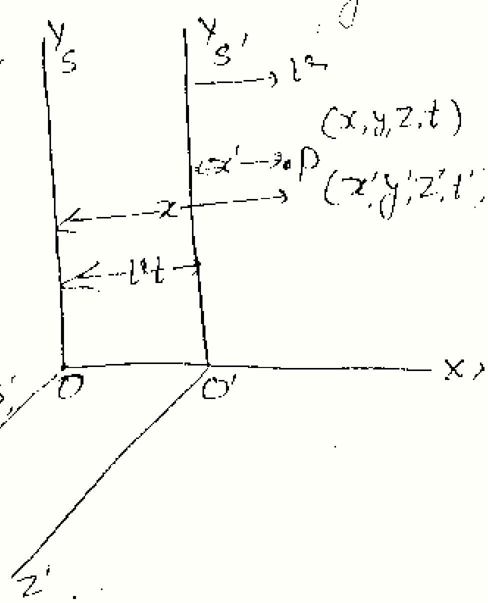
$$c = \frac{r}{t} = \frac{(x^2+y^2+z^2)^{1/2}}{t}$$

$$\Rightarrow x^2+y^2+z^2-c^2t^2 = 0 \quad \text{—(1)}$$

when the event is observed by S', we have

$$x'^2+y'^2+z'^2-c^2t'^2 = 0 \quad \text{—(2)}$$

(c is a Constant)



Moreover $y = y'$ and $z = z'$

From eq (1) & (2), we get.

From eqⁿ ① & ②, we get

$$x^2 - c^2t^2 = x'^2 - c^2t'^2 \quad \text{---} (3)$$

We have to find out the relation between the corresponding co-ordinates of the two systems. The relation between x' & x may be given by

$$x' = k(x - vt) \quad \text{---} (4)$$

where
k is a constant and does not depend upon x & t but may depend upon v.
If the system S is moving w.r.t S' with velocity $-v$ along positive x-axis direction, then

$$x = k'(x' + vt') \quad \text{---} (5)$$

Substituting the value x' from eqⁿ (4), we get

$$x = k'[k(x - vt) + vt']$$

$$\frac{x}{k'} = k(x - vt) + vt'$$

$$vt' = -k(x - vt) + \frac{x}{k'}$$

$$t' = -\frac{k}{v}(x - vt) + \frac{x}{k'v}$$

$$t' = -\frac{kx}{v} + kt + \frac{x}{k'v}$$

$$t' = k\left[t - \frac{x}{v}\left(1 - \frac{1}{kk'}\right)\right] \quad \text{---} (6)$$

Now the equation take (3) and putting the value x' from eqⁿ(4) and t' from eqⁿ(6), we becomes

$$x^2 - c^2t^2 = k^2(x - vt)^2 - c^2k^2\left[t - \frac{x}{v}\left(1 - \frac{1}{kk'}\right)\right]^2 = 0$$

$$x^2 - c^2t^2 - k^2(x^2 + v^2t^2 - 2xvt) + c^2k^2\left(t^2 - \frac{2xt}{v}\left(1 - \frac{1}{kk'}\right) + \frac{x^2}{v^2}\left(1 - \frac{1}{kk'}\right)\right) = 0 \text{---} (7)$$

equation (7) is an identity and hence the coefficient of x & t must be equal to zero separately.

Equating the coefficient of 'xt' to zero

$$2k^2v + c^2k^2\left[-\frac{2}{v}\left(1-\frac{1}{kk'}\right)\right] = 0$$

$$2k^2v + c^2k^2\left[-\frac{2}{v} + \frac{2}{vkk'}\right] = 0$$

On the simplifying the eqn we have

$$2k^2v + \left(-\frac{2c^2k^2}{v}\right) + \frac{2c^2k^2}{vkk'} = 0$$

$$2k\left[kv - \frac{c^2k}{v} + \frac{c^2}{vk'}\right] = 0$$

$$\text{or } kk'v^2 - c^2kk' + c^2 = 0$$

$$\boxed{kk'(v^2-c^2) + c^2 = 0} \quad \text{--- (8)}$$

On equating the coefficient of t^2 to zero, from eqn (7), we

$$-c^2 - k^2v^2 + c^2k^2 = 0$$

$$c^2k^2 - k^2v^2 = c^2$$

$$k^2(c^2 - v^2) = c^2$$

$$k^2 = \frac{c^2}{c^2 - v^2} = \frac{1}{1-\frac{v^2}{c^2}}$$

$$\text{or } k = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{--- (9)}$$

Comparing the eqn (8) & (9), we becomes $k = k'$

Putting the value k from eqn (9), in eqn (4), we have

$$x' = \frac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{--- (10)}$$

This is the Lorentz transformation for space (x, y, z).

Similarly, Putting the value of k in eqn (6), we get

$$t' = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\left[t - \frac{x}{v}\left(1-\frac{1}{k^2}\right)\right] \quad \because k = k'$$

$$t' = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\left[t - \frac{x}{v}\left(1-\frac{c^2-v^2}{c^2}\right)\right] \quad \because k = \frac{c^2}{c^2-v^2}$$

$$t' = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\left[t - \frac{x}{v}\cdot\frac{v^2}{c^2}\right]$$

$$\boxed{t' = \frac{t - \frac{xv}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}} \quad \text{—— (11)}$$

This is Lorentz transformation for time(t).

L.T for space (x, y, z) and time(t) are

$$x' = \frac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}}, \quad y' = y, \quad z' = z \quad \text{and} \quad t' = \frac{t - \frac{xv}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

If the system S is moving with velocity ($-v$) w.r.t to S'. We have getting the inverse L.T eqn by Putting ($-v$) in place (v) and interchanged Primed and unprimed quantities hence the inverse L.T. eqn are

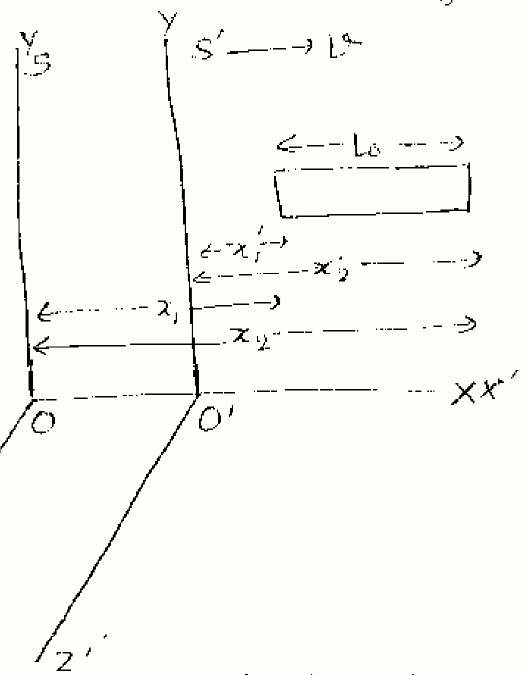
$$x = \frac{x' + vt'}{\sqrt{1-\frac{v^2}{c^2}}}, \quad y = y', \quad z = z' \quad \& \quad t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

Length contraction:

Suppose a body is at rest w.r.t an observer, its length is determined by measuring the difference between the spatial(i.e referring to space) co-ordinates of the end points of the body. Since the body is not moving, these measurements may be made at any time and length so determined is called the rest length or proper length of the body.

Let us consider a frame of reference S' moving with a uniform velocity v relative to stationary frame S in positive direction of x-axis. Let a rigid rod of proper length(L_0) be placed with its length parallel to the x-axis in moving frame S' as shown in fig.

[The proper length is the length of the rod as measured by an observer at rest w.r.t the rod]



Suppose x_1' and x_2' be the co-ordinates of two end points of the rod w.r.t observer in frame S'. The length of the rod L_0 as observed by an observer in moving frame S' at any instant is given by

$$L_0 = x_2' - x_1' \quad \text{——} (1)$$

The length of the rod L measured by an observer in stationary frame of reference S at the same instant will b

$$L = x_2 - x_1 \quad \text{——} \quad (2)$$

where x_1 & x_2 be the co-ordinate of the end Points of the rod in the fixed frame of references

Using Lorentz equations for a given value of t, we hav

$$x_2' = \frac{x_2 - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{and} \quad x_1' = \frac{x_1 - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Subsituting these values of x_1' & x_2' in eqn (1), we ge

$$L_0 = \frac{x_2 - vt}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{x_1 - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$L_0 = \frac{x_2 - x_1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{From eqn (2)}$$

$$L_0 = \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{or} \quad \boxed{L = L_0\sqrt{1 - \frac{v^2}{c^2}}} \text{——} (3)$$

Since $\left(1 - \frac{v^2}{c^2}\right)^{1/2} < 1$, then $L < L_0$

Thus the measured length L of the moving rod along th direction of motion is contracted by a factor $\sqrt{1 - \frac{v^2}{c^2}}$ from its Proper Length; This is also known as Lorentz-Fitzerald contraction.

(i) if $v << c$, then $\frac{v^2}{c^2}$ is negligible

$\therefore L = L_0$

(ii) if $v = c$ then $L < L_0$

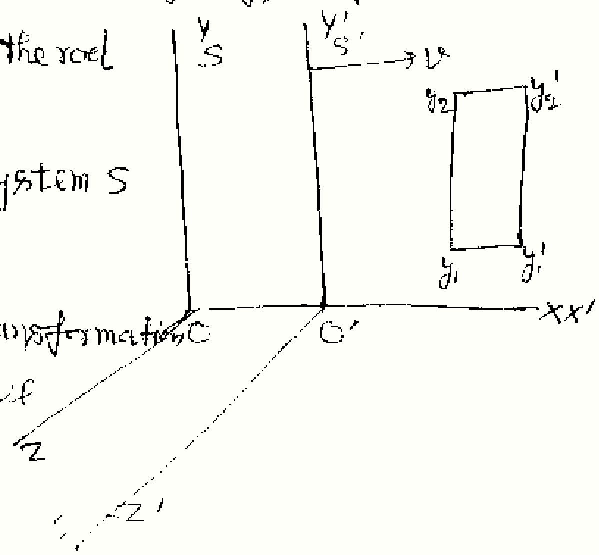
→

(iii) if $v = c$, then $\frac{v^2}{c^2} = 1$
and therefore $L = 0$ which is not possible.

(iv) The contraction takes place only along the direction of motion and remains unchanged in perpendicular direction.

When the position of the rigid rod of y-axis:

If we consider the length of rod along the perpendicular to the direction of motion of frame S' (say along y-axis) and let y_1 & y_2 are co-ordinates of ends of rod relative to system S corresponding to co-ordinate y_1' & y_2' in frame S'.



then rest length of the rod

$$L_{0y} = y_2' - y_1'$$

length of the rod in system S

$$L_y = y_2 - y_1$$

Therefore, from Lorentz transformation $y = y'$, we have

$$y_1 = y_1' \quad \& \quad y_2 = y_2'$$

$$\therefore y_2 - y_1 = y_2' - y_1'$$

i.e. $\boxed{L_{0y} = L_y}$

Therefore the length of rod remains unchanged in a direction perpendicular to the direction of motion.

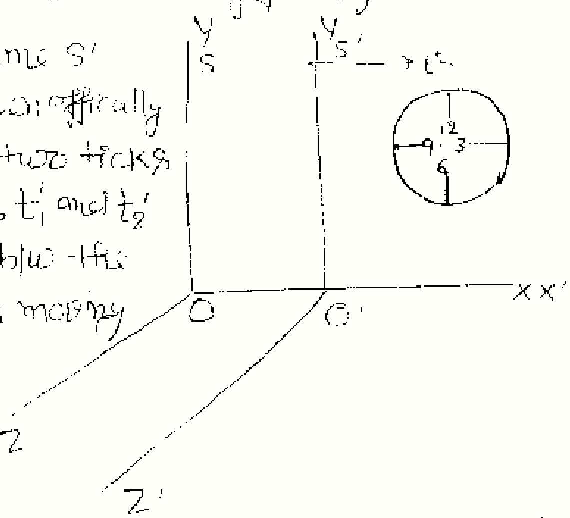
Time - Dilation:

The time interval between two events that occurs at the same place in an observer's frame of reference is called the Proper time of the interval b/w the events.

To derive a relation for time dilation, Let us consider a clock placed at the Position x-axis in moving frame S'. The clock is at rest relative to an observer in S' and is moving with velocity v relative to an observer in frame of reference S (stationary frame)

If an observer in frame S' registered (which has been officially record) the times of two ticks given by the clock as t_1' and t_2' then the time interval b/w the ticks as measured in moving frame S' is given by

$$t_0 = t_2' - t_1' \quad \text{---}(1)$$



where
∴ t_0 is the Proper time interval or the time interval measured in co-ordinate frame in which two events occurred at the same place.

Suppose an observer in stationary frame S, relative to which the clock is moving with velocity v, records the same ticks at times t_1 & t_2. The time interval appears to him is a,

$$t = t_2 - t_1 \quad \text{---} (2)$$

From inverse Lorentz transformation for a formulae of x we have

$$t_2 = \frac{t_2' + \frac{x'v}{c^2}}{\sqrt{1 - v^2/c^2}} \quad \& \quad t_1 = \frac{t_1' + \frac{x'v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Substitute the above values in eq^n ④, we get

$$t = \frac{t_2' + \frac{x'v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{t_1' + \frac{x'v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{t_2' - t_1'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

From eq^n ①,

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{or} \quad \boxed{t_0 = t\sqrt{1 - \frac{v^2}{c^2}}} \text{——} ⑧$$

Since $\sqrt{1 - \frac{v^2}{c^2}} < 1$, then $t > t_0$

This is the relation for Proper interval of time. Eq^n ⑧ shows that the time interval appears to be lengthened (owing to the relative motion) by a factor $\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ to the observer in the stationary frame S. OR a moving clock appears to go slow. This effect is called time dilation.

① if $v = c$, then $\frac{v^2}{c^2} = 1$ this means $t = \infty$. It reveals that a clock moving with the speed of light appears to be completely stopped to an observer in stationary frame of reference.

② If $v << c$, then $\frac{v^2}{c^2}$ is very small & can be neglected.
$\therefore$ $t = t_0$ i.e., if the clock is moving with the speed very much smaller than speed of light, then time interval will remain same ~~for~~ moving observer.

Proper time Interval: The proper time interval is the time interval measured by a clock attached to the observer's body

Time Dilation is Real Effect:-

Take an example of cosmic ray Particle called measons. μ-measons are created at high altitude in the earth atmosphere (= 10 km) by the fast cosmic-ray Photons and are Projected towards the earth surface with a very high speed of about 2.994×10^8 m/sec which is $0.998c$. μ-measons are unstable and decay into electrons or Positrons with an average life time of about 2×10^{-6} sec. Hence, in its life time a μ-meson can travel a distance

$$d = vt = 2.99 \times 10^8 \text{ m/sec} \times 2 \times 10^{-6} = 600 \text{ m}$$

But the Question is that how μ-measons travel a distance of 10 Km to reach the earth surface. This is Possible only because of time dilation.

In its own frame of reference, μ-mesons have an average life time $t_0 = 2.0 \times 10^{-6}$ sec

In observer's frame of reference

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t = \frac{2.0 \times 10^{-6} \text{ sec}}{\sqrt{1 - (0.998)^2}}$$

$$t = \frac{2.0 \times 10^{-6} \text{ sec}}{0.063} = 3.17 \times 10^{-5} \text{ sec}$$

In this dilated life time, a μ-meson can travel a distance

$$d_0 = 2.994 \times 10^8 \text{ m/sec} \times 3.17 \times 10^{-5} \text{ sec}$$

$$= 10 \text{ Km.}$$

This explains the Presence of μ-meson on the earth surface. Hence we can say that time dilation is real ~~effect~~

Addition of Velocities or Velocity Addition Theorem:

The relativistic velocity addition formula can be obtained by using Lorentz Transformation equations as follows:

Suppose a frame of reference S' is moving with a constant velocity v relative to a stationary frame S along the +ve dir of x axis. Let a particle is also moving along the +ve dir of x-axis if the particle moves through a distance dx in a time interval dt in the frame S, then the velocity of a particle as measured by an observer in stationary frame S is given by

$$u = \frac{dx}{dt} \quad \text{---} (1)$$

to an observer in moving frame S', the same distance covered by the particle and the time interval taken will appear different, then the velocity of the particle in moving frame S' will be $u' = \frac{dx'}{dt'}$ --- (2)

From the L.T eqns, we have

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{and} \quad t' = \frac{t - \frac{xv}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

on differentiating the value x' & t', we get

$$dx' = \frac{dx - vdt}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{and} \quad dt' = \frac{dt - \frac{vdx}{c^2}}{\sqrt{1 - v^2/c^2}}$$

Subsituting the values of dx' & dt' in eqn (2), we

$$u' = \frac{dx - vdt}{dt - v\frac{dx}{c^2}}$$

$$u' = \frac{\frac{dx}{dt} - v}{\left(1 - \frac{v}{c^2}\frac{dx}{dt}\right)}$$ From eqn (1)

or $$\boxed{u' = \frac{u - v}{1 - \frac{uv}{c^2}}} \quad \text{—— (3)}$$

This eq^n(3) represents the relativistic velocity addition formula.

The velocity of a Particle in frame S is obtained by inverse L.T obtained by replacing v by $-v$ in eq^n (3) as

$$\boxed{u = \frac{u' + v}{1 + \frac{u'v}{c^2}}} \quad \text{—— (4)}$$

If $u' = c$ i.e the moving particle be a Photon moving with the velocity of light in the +ve direction of x-axis, then its velocity observed by an observer in the from S is given by

$$u = \frac{c + v}{1 + \frac{cv}{c^2}} = \frac{c(c+v)}{(c+v)} = c$$

Thus the observers in frames S' and S, measure the speed of light exactly same.
So the velocity of light (c) is the same in all inertia frames of reference. This is the Second Postulate of Einstein's special theory of relativity.

If we put $u' = c$ and $v = c$, then

From eq^n(4) $\quad u = \dfrac{c + c}{1 + \frac{c^2}{c^2}} = c$

Hence we concluded that the addition of any velocity to the velocity of light simply reproduces the velocity of light.

→ It means that "the velocity of light in vaccum is the maximum attainable velocity in nature and no signal can travel faster than light in vaccum".

Relativistic Velocity transformation:-

From Lorentz transformation equations are;

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad y = y', \quad z = z' \text{ and } t' = \frac{t - \frac{xv}{c^2}}{\sqrt{1 - v^2/c^2}} \quad \text{—(1)}$$

$$\text{and } u_x' = \frac{dx'}{dt'}, \quad u_y' = \frac{dy'}{dt'} \text{ and } u_z' = \frac{dz'}{dt'}$$

Differentiations of eq^n (1) give

$$dx' = \frac{dx - vdt}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad dy = dy', \quad dz = dz' \text{ \& } dt' = \frac{dt - \frac{v}{c^2}dx}{\sqrt{1 - v^2/c^2}}$$

$$\therefore u_x' = \frac{dx'}{dt'} = \frac{dx - vdt}{dt - \frac{v}{c^2}dx}$$

$$u_x' = \frac{\frac{dx}{dt} - v}{1 - \frac{v}{c^2}\frac{dx}{dt}} = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}$$

$$u_y' = \frac{dy'}{dt'} = \frac{dy}{\left(dt - \frac{v}{c^2}dx\right) / \left(1 - \frac{v^2}{c^2}\right)^{1/2}} = \frac{\frac{dy}{dt}\left(1 - \frac{v^2}{c^2}\right)^{1/2}}{1 - \frac{v}{c^2}\frac{dx}{dt}}$$

$$u_y' = \frac{u_y\left(1 - \frac{v^2}{c^2}\right)^{1/2}}{\left(1 - \frac{u_x v}{c^2}\right)}$$

$$\text{Similarly } u_z' = \frac{dz'}{dt'} = \frac{\frac{dz}{dt}\left(1 - \frac{v^2}{c^2}\right)^{1/2}}{1 - \frac{v}{c^2}\frac{dx}{dt}} = \frac{u_z\left(1 - \frac{v^2}{c^2}\right)^{1/2}}{1 - \frac{u_x v}{c^2}}$$

Relativistic Acceleration Transformation:-

Lorentz transformation eq's are

$$x' = \frac{x - ut}{\sqrt{1 - \frac{u^2}{c^2}}}, \; y = y', \; z = z' \text{ and } t' = \frac{t - \frac{xu}{c^2}}{\sqrt{1 - \frac{u^2}{c^2}}}$$

These gives on differentiation

$$dx' = \frac{dx - u\,dt}{\sqrt{1 - \frac{u^2}{c^2}}}, \; dy = dy', \; dz = dz' \;\&\; dt' = \frac{dt - \frac{u}{c^2}dx}{\sqrt{1 - u^2/c^2}}$$

The velocity transformation in moving frame S' are

$$u_x' = \frac{u_x - u}{1 - \frac{u_x u}{c^2}}, \; u_y' = \frac{u_y\left(1 - \frac{u^2}{c^2}\right)^{1/2}}{1 - \frac{u_x u}{c^2}} \;\&\; u_z' = \frac{u_z\left(1 - \frac{u^2}{c^2}\right)^{1/2}}{1 - \frac{u_x u}{c^2}}$$

u_x' on differentiation gives,

$$du_x' = \frac{du_x - 0}{1 - \frac{u_x v}{c^2}} + (u_x - v)(-1)\left(1 - \frac{u_x v}{c^2}\right)^{-2}\left(-\frac{v}{c^2}\right)du_x$$

$(\because v \text{ is Constant})$

$$du_x' = \frac{du_x}{\left(1 - \frac{u_x v}{c^2}\right)} + \frac{(u_x - v)\frac{u}{c^2}\,du_x}{\left(1 - \frac{u_x v}{c^2}\right)^2}$$

$$\therefore \text{Acceleration } a_x' = \frac{du_x'}{dt'} = \frac{du_x \Big/ \left(1 - \frac{u_x u}{c^2}\right)}{\left(dt - \frac{u\,dx}{c^2}\right)\Big/\left(1 - \frac{u^2}{c^2}\right)^{1/2}} + \frac{(u_x - u)\frac{v}{c^2}du_x\Big/\left(1 - \frac{u}{c}\ldots\right)}{\longrightarrow \;\; ''}$$

$$\text{or } a_x' = \frac{\frac{du_x}{dt}\sqrt{1 - \frac{u^2}{c^2}}}{\left(1 - \frac{u}{c^2}\frac{dx}{dt}\right)\left(1 - \frac{u_x v}{c^2}\right)} + \frac{(u_x - v)\frac{v}{c^2}\frac{du_x}{dt}\left(1 - u^2/c^2\right)^{1/2}}{\left(1 - \frac{u}{c^2}\frac{dx}{dt}\right)\left(1 - \frac{u_x u}{c^2}\right)^2}$$

$$\text{or } a_x' = \frac{a_x\left(\sqrt{1 - u^2/c^2}\right)}{\left(1 - \frac{u_x v}{c^2}\right)^2} + \frac{(u_x - v)\frac{u}{c^2}a_x\sqrt{1 - u^2/c^2}}{\left(1 - \frac{u_x v}{c^2}\right)^3}$$

$$\text{or } a_x' = \frac{a_x\sqrt{1 - v^2/c^2}\left[1 - \frac{u_x u}{c^2} + \frac{u_x u}{c^2} - \frac{u^2}{c^2}\right]}{\left(1 - \frac{u_x u}{c^2}\right)^3}$$

$$\text{or } a_x' = \frac{a_x\left(1-\frac{u^2}{c^2}\right)^{3/2}}{\left(1-\frac{u_x u}{c^2}\right)^3}$$

Similarly

$$du_y' = \left(1-\frac{u^2}{c^2}\right)^{1/2}\left\{d\left[u_y\Big/\left(1-\frac{u_x u}{c^2}\right)\right]\right\}$$

$$= \left(1-\frac{u^2}{c^2}\right)^{1/2}\left\{\frac{du_y}{1-\frac{u_x u}{c^2}} + (-1)\,u_y\left(1-\frac{u_x u}{c^2}\right)^{-2}\left(-\frac{u}{c^2}\right)du_x\right\}$$

$$= \left(1-\frac{u^2}{c^2}\right)^{1/2}\left\{\frac{du_y}{1-\frac{u_x u}{c^2}} + \frac{\frac{u_y u}{c^2}\,du_x}{\left(1-\frac{u_x u}{c^2}\right)^2}\right\}$$

$$a_y' = \frac{du_y'}{dt'} = \frac{\left(1-\frac{u^2}{c^2}\right)^{1/2}\left\{\left[du_y\Big/\left(1-\frac{u_x u}{c^2}\right)\right] + \frac{u_y u}{c^2}du_x\Big/\left(1-\frac{u_x u}{c^2}\right)^2\right\}}{\left(dt-\frac{u}{c^2}dx\right)\Big/\left(1-\frac{u^2}{c^2}\right)^{1/2}}$$

$$= \left(1-\frac{u^2}{c^2}\right)\left\{\frac{du_y/dt}{\left(1-\frac{u}{c^2}\frac{dx}{dt}\right)\left(1-\frac{u_x u}{c^2}\right)} + \frac{\frac{u_y u}{c^2}\,\frac{du_x}{dt}}{\left(1-\frac{u}{c^2}\frac{dx}{dt}\right)\left(1-\frac{u_x u}{c^2}\right)^2}\right\}$$

$$a_y' = \left(1-\frac{u^2}{c^2}\right)\left[\frac{a_y}{\left(1-\frac{u_x u}{c^2}\right)^2} + \frac{\frac{u_y u}{c^2}\,a_x}{\left(1-\frac{u_x u}{c^2}\right)^3}\right]$$

Similarly

$$a_z' = (\,\,)\left[\frac{a_z}{\left(1-\frac{u_x u}{c^2}\right)^2} + \frac{\frac{u_z u}{c^2}\,a_x}{\left(1-\frac{u_x u}{c^2}\right)^3}\right]$$

For a particle instantaneous at rest in S frame, $u_x = u_y = u_z = 0$ and we get

$$a_x' = \left(1-\frac{u^2}{c^2}\right)^{3/2}a_x,\quad a_y' = \left(1-\frac{u^2}{c^2}\right)a_y \text{ and } a_z' = \left(1-\frac{u^2}{c^2}\right)a_z$$

Variation of mass with velocity :-

According to the classical idea, the inertial mass of a body is independent of its velocity and is a constant under all circumstances. In relativistic mechanics the mass of a body varies with its velocity.

"The mass of a body moving at high speed ($v \approx c$) relative to an observer is larger than its mass when it is rest by a factor $1/\sqrt{1-\frac{u^2}{c^2}}$."

Using law of conservation of linear momentum together with Lorentz Transformation equations the following expression for the variation in mass with velocity is obtained.

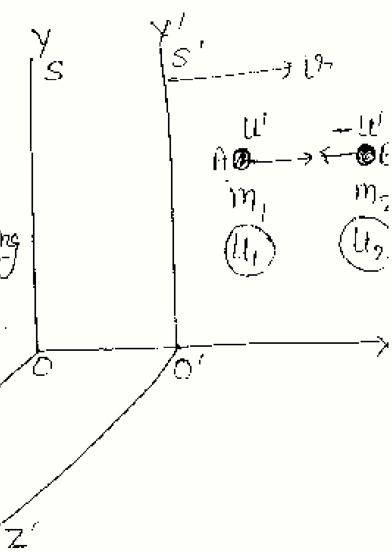
$$m = \frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}}$$

where m_0 is the rest mass of the body, $m \rightarrow$ is the measured mass of the body when it is moving w.r.t an observer and is known as relativistic mass. Relativistic mass increase is significant only at speeds approaching to that of light.

Derivation :-

The variation of mass with velocity can be derived by using relativistic invariance of the law of conservation of momentum as follows:

Let us consider two inertial frames of reference S and S'. S is at rest while S' is moving with constant velocity v w.r.t S along the +ve direction of x-axis. Let two identical bodies A and B of masses m_1 & m_2 are moving with velocities u' & $-u'$ respectively.



in the moving frame s' along a straight line as shown in a fig. Let us consider the collision between the two bodies w.r. the frame s. if u_1 & u_2 be the velocities the two bodies A & B relative to an observer in stationary frame S the According to the laws of addition of relativistic velocit

$$u_1 = \frac{u' + v}{1 + \frac{u'v}{c^2}} \quad \text{---} \ (1)$$

$$u_2 = \frac{-u' + v}{1 - \frac{u'v}{c^2}} \quad \text{---} \ (2)$$

At the time of collision, the two bodies are momentar at rest relative to the frame S', But as seen the frame S with moving velocity v. Since the momentum of a body is an invariant quantity.

Therefore, according to the Principle of conservation of momen

momentum before impact = momentum after impact

$$m_1 u_1 + m_2 u_2 = (m_1 + m_1) v \quad \text{---} \ (3)$$

Substituting the value u_1 & u_2 in eq^n (3), we get

$$m_1 \left[\frac{u' + v}{1 + \frac{u'v}{c^2}} \right] + m_2 \left[\frac{-u' + v}{1 - \frac{u'v}{c^2}} \right] = m_1 v + m_2 v$$

or

$$m_1 \left[\frac{u' + v}{1 + \frac{u'v}{c^2}} - v \right] = m_2 \left[v - \frac{-u' + v}{1 - \frac{u'v}{c^2}} \right]$$

$$\text{or}\quad m_1\left[\frac{u'+v-v-\frac{u'v^2}{c^2}}{1+\frac{u'v}{c^2}}\right] = m_2\left[\frac{v-\frac{u'v^2}{c^2}+u'-v}{1-\frac{u'v}{c^2}}\right]$$

$$m_1\left(\frac{u'-\frac{u'v^2}{c^2}}{1+\frac{u'v}{c^2}}\right) = m_2\left[\frac{u'-\frac{u'v^2}{c^2}}{1-\frac{u'v}{c^2}}\right]$$

$$\text{or}\quad \frac{m_1}{m_2} = \frac{1+\frac{u'v}{c^2}}{1+\frac{(u')v}{c^2}} \quad \text{---} \quad (4)$$

Now squaring eqn (1), we have

$$u_1^2 = \left(\frac{u'+v}{1+\frac{u'v}{c^2}}\right)^2$$

$$\text{or}\quad 1-\frac{u_1^2}{c^2} = 1-\frac{1}{c^2}\cdot\frac{(u'+v)^2}{\left(1+\frac{u'v}{c^2}\right)^2}$$

$$= \frac{\left(1+\frac{u'v}{c^2}\right)^2-\frac{1}{c^2}(u'+v)^2}{\left(1+\frac{u'v}{c^2}\right)^2}$$

$$= \frac{1+\frac{u'^2v^2}{c^4}+\frac{2u'v}{c^2}-\frac{u'^2}{c^2}-\frac{v^2}{c^2}-\frac{2u'v}{c^2}}{\left(1+\frac{u'v}{c^2}\right)^2}$$

$$= \frac{\left(1-\frac{v^2}{c^2}\right)-\frac{u'^2}{c^2}\left(1-\frac{v^2}{c^2}\right)}{\left(1+\frac{u'v}{c^2}\right)^2}$$

$$1-\frac{u_1^2}{c^2} = \frac{\left(1-\frac{v^2}{c^2}\right)\left(1-\frac{u'^2}{c^2}\right)}{\left(1+\frac{u'v}{c^2}\right)^2}$$

$$\left(1+\frac{u'u}{c^2}\right)^2 = \frac{\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{u'^2}{c^2}\right)}{\left(1+\frac{u_1^2}{c^2}\right)^2}$$

or $$1+\frac{u'v}{c^2} = \sqrt{\frac{\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{u'^2}{c^2}\right)}{\left(1-\frac{u_1^2}{c^2}\right)}} \quad \text{---} (5)$$

Similarly
$$1-\frac{u'u}{c^2} = \sqrt{\frac{\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{u'^2}{c^2}\right)}{\left(1-\frac{u_2^2}{c^2}\right)}} \quad \text{---} (6.)$$

Putting the value ~~from~~ eq^n (5) & (6) in eq^n (4), we get

$$\frac{m_1}{m_2} = \sqrt{\frac{1-\frac{u_2^2}{c^2}}{1-\frac{u_1^2}{c^2}}} \quad \text{---} (7.)$$

if the body B is at rest or moving with zero velocity in stationary frame S i.e $u_2 = 0$ before collision.
then we have take $m_2 = m_0$ the rest mass of the bo

$$\frac{m_1}{m_0} = \frac{1}{\sqrt{1-\frac{u_1^2}{c^2}}}$$

or $$m_1 = \frac{m_0}{\sqrt{1-\frac{u_1^2}{c^2}}} \quad \text{---} (8.)$$

Suppose we have consider a single body is at res
mass (m_0) and its take the velocity $u_1 = v$, then the

relativistic mass formula may be

$$\boxed{m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}} \quad \text{------} \quad (9)$$

This is the required relativistic formula for the variation of mass with velocity.

Conclusions:

(i) As the velocity v of the Particle relative to the obser increases, the mass of the Particle increases.

(ii) when $v << c$, then $\frac{v^2}{c^2}$ is very small and may be negl-ed compared to (i) then $m \approx m_0$ (at ordinary velocities the two masses are equal)

(iii) when $v = c$, $m = \infty$ if a material particle trave with the speed of light, its mass would become infi which is impossible.

Experimental Verification:-

The first confirmation of eqn. (9) was discovery by Bucher in 1908 that the ratio $\frac{e}{m}$ of the electron's charge to its mass is smaller for fast electrons than the slow ones. Since then, it has been verified by a number of experiments.

Relativistic Momentum and Force: In relativistic mecha the Newton's second law, when the force (F) is para velocity ($\vec{v}$), i.e. the force acting on a Particle can not be defined as the Product of mass and acceleration of the Part but as the time rate of change of its momentum.

$$\text{Thus } F = \frac{dP}{dt} = \frac{d}{dt}(mv) \quad \{\because P = mv$$

In relativistic mechanics,

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\vec{F} = \frac{d}{dt}\left[\frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}\right] \quad \text{--- (10)}$$

Ex for: show that $\vec{F} = m_0 \frac{d\vec{v}}{dt}\left(1-\frac{v^2}{c^2}\right)^{-3/2}$

Proof: From eq^n (10), we have

$$\vec{F} = m_0 \frac{d}{dt}\left[v \cdot \left(1-\frac{v^2}{c^2}\right)^{-1/2}\right]$$

$$= m_0\left[\frac{dv}{dt}\left(1-\frac{v^2}{c^2}\right)^{-1/2} + v \cdot \frac{d}{dt}\left(1-\frac{v^2}{c^2}\right)^{-1/2}\right]$$

$$= m_0\left[\frac{dv}{dt}\left(1-\frac{v^2}{c^2}\right)^{-1/2} + v \times \left(-\frac{1}{2}\right)\left(1-\frac{v^2}{c^2}\right)^{-3/2} \cdot -\frac{2v}{c^2}\frac{dv}{dt}\right]$$

$$= m_0\left[\frac{dv}{dt}\left(1-\frac{v^2}{c^2}\right)^{-1/2} + \frac{v^2}{c^2}\frac{dv}{dt}\left(1-\frac{v^2}{c^2}\right)^{-3/2}\right]$$

$$= m_0 \frac{dv}{dt}\left(1-\frac{v^2}{c^2}\right)^{-3/2}\left[1-\frac{v^2}{c^2}+\frac{v^2}{c^2}\right]$$

$$\boxed{\vec{F} = m_0 \frac{dv}{dt}\left(1-\frac{v^2}{c^2}\right)^{-3/2}}$$

This is the obtained relativistic form of Newton's second law.

Mass - Energy - Equivalence (Einstein's Mass-Energy Relation)

This equation ($E = mc^2$) is most famous and most significa relationship obtained by Einstein from the postulates of their special theory of relativity. Numerous experiments have demonstrated that "the mass is convertible into energy and energy into mass, and that the conversion factor between two is the square of speed of light. The mass-energy equivalence can be deduced directly from special theory of relativity as follows.

Let us consider the force F acting on a body of mass(m) in the direction of its velocity

if the force F displaces the body by a distance ds, then by work-energy theorem, the work done by the force dw will get stored in the body as its kinetic Energy

i.e $\quad dK = dW = F\,ds \quad \text{--- (1)}$

According to Newton's law of motion in relativistic mechani the force is the rate of change of momentum.

$$F = \frac{dp}{dt} = \frac{d}{dt}(mv) \qquad \{\because p = mv\}$$

Acc. to the theory of relativity, mass varies with velocity

$$F = m\frac{dv}{dt} + v\frac{dm}{dt} \quad \text{--- (2)}$$

Substituting the value F in eqn (1), we get:

$$dK = \left(m\frac{dv}{dt} + v\frac{dm}{dt}\right) \cdot ds$$

$$dK = m\frac{dv}{dt}ds + v\frac{dm}{dt}ds$$

$$= m\frac{ds}{dt}dv + v\frac{ds}{dt}dm$$

$$dK = mv\,dv + v^2 dm \quad \text{--- (3)}$$

Acc. to the special theory of relativity, mass varies with velocity, we have

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = m_0\left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

differentiating above it

$$dm = m_0\left(-\frac{1}{2}\right)\left(1 - \frac{v^2}{c^2}\right)^{-3/2}\left(-\frac{2v}{c^2}\right)dv$$

$$dm = \frac{m_0 v\, dv}{c^2(1 - v^2/c^2)^{3/2}}$$

$$dm = \frac{m(1 - v^2/c^2)^{1/2} v\, dv}{c^2(1 - v^2/c^2)^{3/2}} = \frac{mv}{c^2\left(1 - \frac{v^2}{c^2}\right)} dv$$

or $mv\, dv = c^2\left(1 - \frac{v^2}{c^2}\right)dm$

or $mv\, dv = (c^2 - v^2)\, dm$ Putting this value in eq

$dK = (c^2 - v^2)dm + v^2 dm$

or $dK = c^2 dm$ integrating m_0 to m, we get

$$K = c^2 \int_{m_0}^{m} dm$$

$$K = c^2(m - m_0) \quad \text{--- (4)}$$

This is the required relativistic Kinetic Energy (K) of a Particle. It is clear that from this expression, the increase in K.E is due to the increase in mass of the particle on account of its relative motion and is equal to the Product of gain in mass and square of the velocity of light.

The total energy of a moving particle is the sum of K.E of motion and rest energy, is

Total energy, E = Rest energy + relativistic K.E

$= m_0c^2 + (m - m_0)c^2$

$$\boxed{E = mc^2} \quad \text{--- (5)}$$

This eqn (4) is well known Einstein mass-energy relation.

$$\therefore \boxed{E = E_0 + K.E} \quad \text{--- (6)}$$

Total energy $$\boxed{E = \frac{E_0}{\sqrt{1-\frac{u^2}{c^2}}}} \quad \text{--- (7)}$$

Limiting Value of Relativistic Kinetic Energy:-

The expression for relativistic K.E is

$$K = c^2(m - m_0) = \left[\frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}} - m_0\right] c^2$$

$$K = m_0 c^2 \left[\left(1-\frac{u^2}{c^2}\right)^{-1/2} - 1\right]$$

Expanding using binomial theorem, we get

$$K = m_0 c^2 \left[1 + \frac{1}{2}\frac{u^2}{c^2} + \frac{3}{8}\frac{u^4}{c^4} + \cdots - 1\right]$$

if $u << c$, i.e $\frac{u}{c} << 1$, then higher terms may be neglected

Thus $$K = m_0 c^2 \left[1 + \frac{u^2}{2c^2} - 1\right] = \frac{1}{2} m_0 u^2$$

$$\boxed{K = \frac{1}{2} m_0 u^2}$$

This is well known classical expression for K.E of the Particle moving with velocity (u). In relativistic mechanics, does reduce to classical mechanics in the limits of small velocities. Fig. shows that how the K.E of a moving object varies with its speed according to both the classical & relativistic mechanics

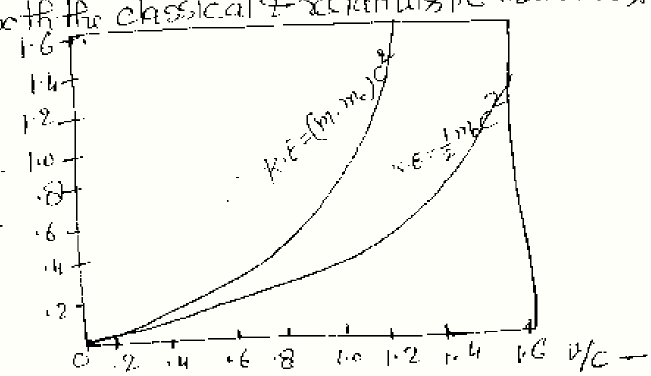


Fig- A comparison b/w the classical and relativistic formulas for the ratio b/w K.E of a moving body and its rest energy m_0c^2.

Relativistic Relation between Energy and Momentum:-

The relativistic total energy of a particle moving with velocity u is given as $E = mc^2 = \frac{m_0 c^2}{\sqrt{1-\frac{u^2}{c^2}}}$ — (1)

where m_0 is the rest mass of the particle.

The momentum of the particle, $p = mu$

or $u = \frac{p}{m}$ — (2)

Substituting the value u in eqn (1), we get

$$E = \frac{m_0 c^2}{\sqrt{1-\frac{p^2}{m^2c^2}}} = \frac{m_0 c^2}{\sqrt{1-\frac{p^2c^2}{(mc^2)^2}}} \quad \because E = mc^2$$

$$E = \frac{m_0 c^2}{\sqrt{1-\frac{p^2c^2}{E^2}}}$$ Squaring the both side, we have

$$E^2 = \frac{m_0^2 c^4}{1-\frac{p^2c^2}{E^2}} \quad \text{or} \quad E^2\left(1-\frac{p^2c^2}{E^2}\right) = m_0^2 c^4$$

or $E^2 - p^2c^2 = m_0^2 c^4$

or $\boxed{E = \sqrt{m_0^2 c^4 + p^2c^2}}$ — (3) This is the relativistic relation b/w the total energy (E) & mom (P) of a particle.

Conclusion:

1. $E^2 - p^2c^2$ is invariant under Lorentz transformation because m_0 and c both are constant.
2. For $u \ll c$, then $E = \frac{p^2}{2m_0}$
3. For massless particle (like Photon)

$m_0 = 0$

$\therefore \boxed{E = pc}$

Relativistic Relation between K.E and Momentum

The expression for the relativistic K.E (K) of a Particle of rest mass m_0 is given by

$$K = E - m_0c^2 \quad \text{But } E = \sqrt{m_0^2c^4 + p^2c^2}$$

$$\therefore K = \sqrt{m_0^2c^4 + p^2c^2} - m_0c^2$$

$$\text{or} \quad K = \left[\left(1 + \frac{p^2}{m_0^2c^2}\right)^{1/2} - 1\right] m_0c^2 \quad \text{—— (1)}$$

This is the obtained relativistic relation b/w K.E & mom of a Particle.

If $v \lt\lt c$, i.e $p \lt\lt m_0c^2$ then

$$K = m_0c^2\left[\left(1 + \frac{1}{2}\frac{p^2}{m_0^2c^2} + \cdots\right) - 1\right]$$

Neglected the higher term, we get

$$K = \frac{p^2}{2m_0}$$

As $v \lt\lt c$, then $m_0 = m$

$$\therefore K = \frac{p^2}{2m}$$

Therefor, for small velocities, the relativistic K E express reduces to classical expression.

Massless Particle :-

A Particle which has zero rest mass (m_0) is called a mass less Particle. In classical Physics, the existence of mass less Particle is impossible, However, in relativistic mechanics, a Particle with zero rest mass can exist.

The relativistic total energy E of a Particle of rest mass (in terms of its momentum p may be expressed as

$$E = \sqrt{m_0^2c^4 + p^2c^2} \quad \text{—— (i)}$$

For massless Particle, $m_0 = 0$

$$\therefore E = PC \quad \text{or} \quad P = \frac{E}{C}$$

Since P is also equal/ $mv = \frac{E}{C} = \frac{mc^2}{c}$ therefore $v = c$ i.e the velocity of massless Particle is same as that of light in free ~~space~~ space.

$\therefore$ The energy of the Particle $E = PC = mc^2$

where m is the mass equivalent to energy, i.e $m = \frac{E}{C^2}$.

Thus we concluded that every massless Particle has energy PC and momentum E/c and moves with the velocit of light(c). The mass of massless Particle is equal to E/c^2. It means that a massless Particle has mass so long as it i in motion. On being stopped they cease to exist, they are either absorbed completely or are changed into ~~other~~ heat at t surface. Thus, the massless particle having energy and moment ,and can exist only when they move with the velocity of light. The Photons and neutrinos are the best examples.