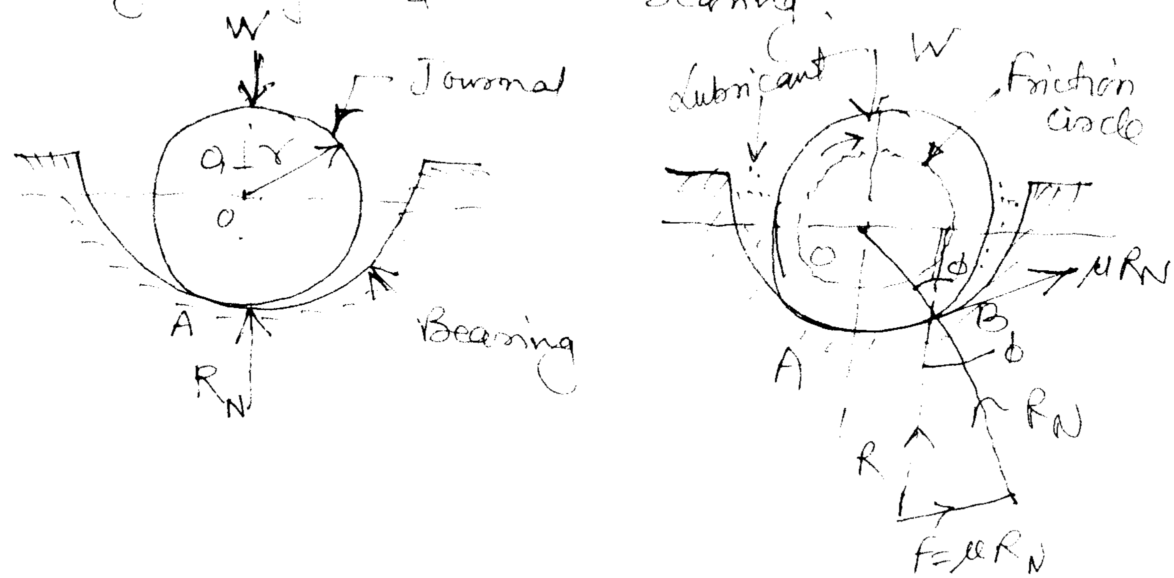
Unit-III

Friction in Journal Bearing – "Friction Circle":

Journal Bearing forms a turning pair as shown in Fig. The fixed outer element of a turning pair is called a 'bearing' and that portion of the inner element which fits in the bearing is called a 'journal'. Journal is slightly less in diameter than the bearing, ~~made~~ in order to permit the free movement of the journal in a bearing.



When the bearing is not lubricated (or journal is stationary), then there is a line contact between the two elements. The load W on the journal & normal reaction R_N of the bearing acts through the centre. The Reaction R_N acts vertically upwards at point A. This point A is known as 'seat' or 'point of pressure'.

Now consider a shaft rotating inside a bearing in clockwise direction. The lubricant between the journal and bearing forms a thin layer which gives reaction. The reaction R acts at another point of pressure B. This is due to the fact that when shaft rotates, a frictional force $F=\mu R_N$ acts at the circumference of the shaft which has tendancy to rotate the shaft in opposite direction of motion, and this shifts the point A to point B.

Let ϕ = Angle between R & R_N

μ = coefficient of friction between journal & bearing

T = frictional torque in N-m.

r = radius of shaft in metres.

Resultant force acting on the shaft must be zero.

$R = W$,

and $T = W \times OC = W \times OB \sin\phi$

$= W \times r \sin\phi$

$\because$ ϕ is very small, therefore $\sin\phi = \tan\phi$

$T = W r \tan\phi = \mu W r$ $\quad (\because \mu = \tan\phi)$

if the shaft rotates with angular velocity ω rad/s, then power wasted in friction,

$$P = T.\omega = T \cdot \frac{2\pi N}{60} \text{ watts.}$$

$$\boxed{P = \frac{2\pi N T}{60} = \frac{2\pi \mu W r N}{60}}$$

The circle which is drawn with centre O and radius $OC = r \sin\phi$ then this circle is called the "friction circle" of a bearing.

Example 10.5: A 60 mm diameter shaft running in a bearing carries a load of 2000 N. if the coefficient of friction between the shaft and bearing is 0.03, find the power transmitted when it runs at 1440 rpm.

Solution:- Given, $d = 60$ mm. or $r = 0.03$ m. $W = 2000$ N.
$\mu = 0.03$; $N = 1440$ rpm. or $\omega = 2\pi \times 1440/60 = 150.8$ rad/s.

We know, $T = \mu W r = 0.03 \times 2000 \times 0.03 = 1.8$ N-m

Power transmitted, $P = T.\omega = 1.8 \times 150.8$

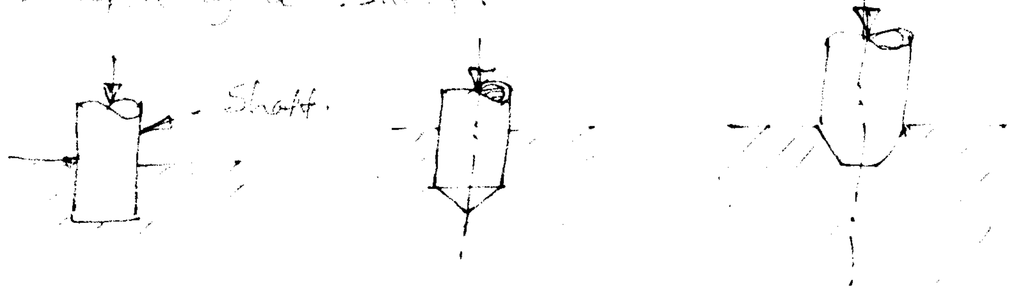
$P = 271.4$ W Ans.

Friction of Pivot and Collar Bearing:—

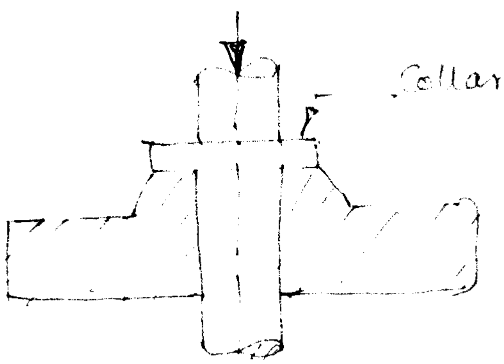
The rotating shafts are frequently subjected to axial thrust. The bearing surfaces such as pivot and collar bearings are used to take this axial thrust of the rotating shaft.

The bearing surfaces placed at the end of a shaft to take the axial thrust are known as 'pivots'.

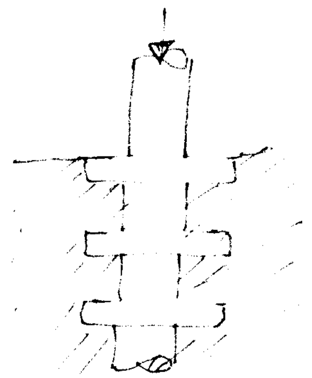
The collar may have flat bearing surface or conical surface. There may be a single collar or several collars bearing along the length of a shaft.



Flat pivot — Conical Pivot — Truncated Pivot

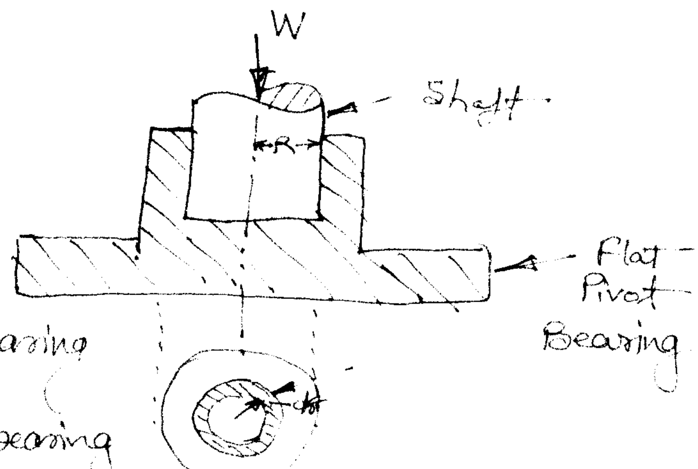


Single flat collar



Multiple flat collars

Flat Pivot Bearing:—



When a vertical shaft rotates in a flat pivot bearing (known as foot step bearing). The sliding friction will be along the surfaces of contact between the shaft and bearing.

Let W = Load transmitted over the bearing surfaces.

of bearing surface between rubbing surfaces.

μ = Coefficient of friction.

There are following two cases :—

1. Considering Uniform pressure :

When the pressure is uniformly distributed over the bearing area, then

$$p = \frac{W}{\pi R^2}$$

Consider a ring of radius r and thickness dr of the bearing area, then

$\cancel{\frac{W}{\pi R^2}}$ Area of bearing surface, $A = 2\pi r\,dr$

Load transmitted to the ring,

$$\delta W = p \times A = p \times 2\pi r\,dr$$

Frictional resistance to sliding on the ring acting tangentially at radius r,

$$F_r = \mu\,\delta W = \mu \times p \times 2\pi r\,dr = 2\pi \mu p r\,dr$$

Frictional Torque on the ring,

$$T_r = F_r \times r = 2\pi \mu p r\,dr \times r$$

$$T_r = 2\pi \mu p r^2\,dr$$

Integrating this equation with in the limits from 0 to R for the total frictional torque on the pivot bearing.

$$\text{Total frictional Torque, } T = \int_0^R 2\pi \mu p r^2\,dr = 2\pi \mu p \int_0^R r^2\,dr$$

$$= 2\pi \mu p \left[\frac{r^3}{3}\right]_0^R = 2\pi \mu p \times \frac{R^3}{3}$$

$$T = \frac{2}{3}\pi \mu p R^3$$

$$\boxed{T = \frac{2}{3}\pi \mu \times \frac{W}{\pi R^2} \times R^3 = \frac{2}{3}\mu W R}$$

When the shaft rotates at ω rad/s, then power lost in friction,

$$P = T \cdot \omega = T \times 2\pi N/60$$

$$P = 2\pi N T$$

2. Considering Uniform wear :—

rate of wear depends upon the intensity of Pressure (p) and velocity of rubbing surfaces (v). It is assumed that the rate of wear is proportional to the product of intensity of Pressure and the velocity of rubbing surfaces (pv)

$r \propto v$

Uniform wear, $p \cdot r = C$ (constant)

$$\text{or } p = \frac{C}{r}$$

Load transmitted to the bearing,

$$\delta W = p \times 2\pi r \, dr$$

$$= \frac{C}{r} \times 2\pi r \, dr = 2\pi C \, dr$$

Total load transmitted to the bearing,

$$W = \int_0^R 2\pi C \cdot dr = 2\pi C \left[r \right]_0^R$$

$$= 2\pi C R \quad \text{or} \quad C = \frac{W}{2\pi R}$$

as we know, Frictional torque acting on the ring,

$$T_r = 2\pi \mu p r^2 \, dr = 2\pi \mu \times \frac{C}{r} \times r^2 \, dr$$

$$T_r = 2\pi \mu C r \, dr$$

Total friction torque on the bearing,

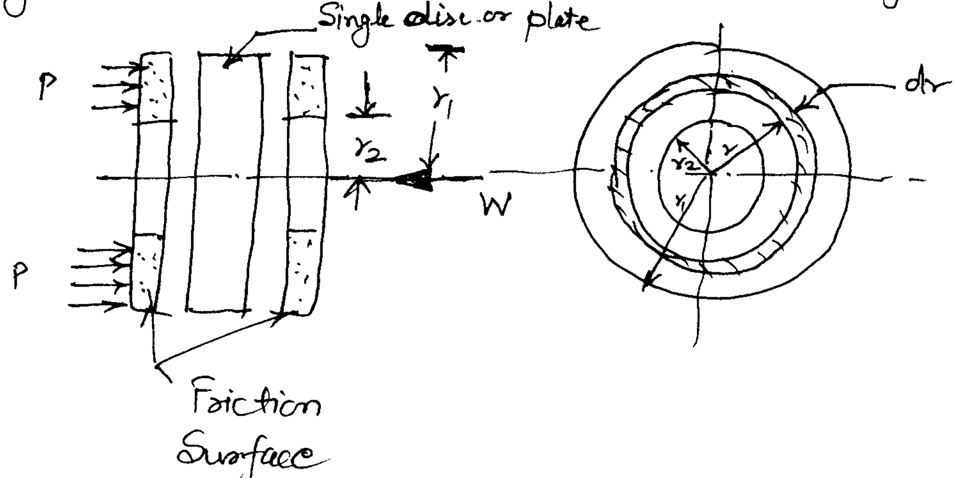
$$T = \int_0^R 2\pi \mu C r \, dr = 2\pi \mu C \left[\frac{r^2}{2} \right]_0^R$$

$$= 2\pi \mu C \times \frac{R^2}{2} = \pi \mu C R^2$$

$$\boxed{T = \pi \mu \times \frac{W}{2\pi R} \times R^2 = \frac{1}{2} \mu W R}$$

Single Disc or Plate clutch :—

Consider two friction surfaces, maintained in Contact by an axial thrust W as shown in Fig.



Let T = Torque transmitted by the clutch.

p = Intensity of Axial pressure with which the Contact Surfaces are held together.

r_1 and r_2 = External and Internal radii of friction faces, and

μ = Coefficient of friction.

Consider an elementry ring of radius r and thickness dr

area of contact Surface or friction Surfaces.

$= 2\pi r\, dr$

Normal or axial force on the ring,

$$\delta W = \text{Pressure} \times \text{Area} = p \times 2\pi r\, dr$$

Frictional Force on the ring acting tangentially at radius r,

$$F_r = \mu \cdot \delta W = \mu \cdot p \times 2\pi r\, dr$$

$\therefore$ Frictional Torque acting on the ring,

$$T_r = F_r \times r = \mu\, p\, 2\pi r\, dr \times r$$

$$T_r = 2\pi \mu p r^2 dr$$

There is are two type of case :—

1. Uniform pressure
2. Uniform Wear

1. Considering Uniform pressure :—

When the pressure is uniformly distributed over the entire area of the friction ~~force~~ face, then the Intensity of Pressure,

$$p = \frac{W}{\pi[r_1^2 - r_2^2]}$$

Frictional Torque, $T_r = 2\pi \mu p r^2 dr$.

Total frictional torque acting on the friction surface or on the clutch,

$$T = \int_{r_1}^{r_2} 2\pi \mu p r^2 dr = 2\pi \mu p \left[\frac{r^3}{3}\right]_{r_1}^{r_2} = 2\pi \mu p \left[\frac{r_1^3 - r_2^3}{3}\right]$$

$$T = 2\pi \mu \times \frac{W}{\pi[r_1^2 - r_2^2]} \times \frac{r_1^3 - r_2^3}{3}$$

$$T = \frac{2}{3}\pi \mu W \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right] = \frac{2}{3}\pi \mu W R$$

where R = Mean radius of friction surface.

$$R = \frac{2}{3}\left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

2. Considering Uniform wear :—

$$p \cdot r = C \quad \text{or} \quad p = C/r$$

Normal force on the ring, $\delta W = p \times 2\pi r dr$

$$= \frac{C}{r} \cdot 2\pi r \, dr$$

$$\delta W = 2\pi C \, dr$$

Total force acting on the friction surfaces,

$$W = \int_{r_2}^{r_1} 2\pi C \cdot dr = 2\pi C[r]_{r_2}^{r_1} = 2\pi C(r_1 - r_2)$$

or $$C = \frac{W}{2\pi(r_1 - r_2)}$$

Frictional torque acting on the ring,

$$T_r = 2\pi\mu p r^2 dr = 2\pi\mu \times \frac{C}{r} \times r^2 dr$$
$$= 2\pi\mu C r\, dr$$

Total frictional torque on the friction surface,

$$T = \int_{r_2}^{r_1} 2\pi\mu C r\, dr = 2\pi\mu C \left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$
$$= 2\pi\mu C \left(\frac{r_1^2 - r_2^2}{2}\right)$$
$$= \pi\mu C \left[r_1^2 - r_2^2\right] = \pi\mu \times \frac{W}{2\pi(r_1 - r_2)} \times \left[r_1^2 - r_2^2\right]$$
$$= \frac{1}{2}\mu W (r_1 + r_2) = \mu W R$$

where, R = Mean radius of the friction surface $= \dfrac{r_1 + r_2}{2}$

Notes: 1. The Total frictional torque acting on the friction surface is given by

$$T = n\mu W R$$

where n = No. of Pairs of friction on Contact Surface.

R = mean radius of friction surface.

$$R = \frac{2}{3}\left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right] \quad \text{for Uniform Pressure}$$

$$= \frac{r_1 + r_2}{2} \quad \text{for Uniform wear}$$

2. for Single disc or plate clutch $n = 2$

3. $P_{max.} \times r_2 = C$ intensity of pressure is max. at outer radius.

4. $P_{min} \times r_1 = C$

5. $P_{av.} = \dfrac{W}{\pi(r_1^2 - r_2^2)}$

Multiple Disc Clutch:—

Multiple disc clutch may be used when a large torque is to be transmitted. The multiple disc clutches are extensively used in motor cars, machine tools etc.

Let n_1 = No. of discs on the driving shaft.

n_2 = No. of discs on the driven shaft

∴ No. of pairs of contact surfaces,

$$n = n_1 + n_2 - 1$$

Total frictional Torque acting on the friction surfaces or on the clutch,

$$T = n \cdot \mu \cdot W \cdot R$$

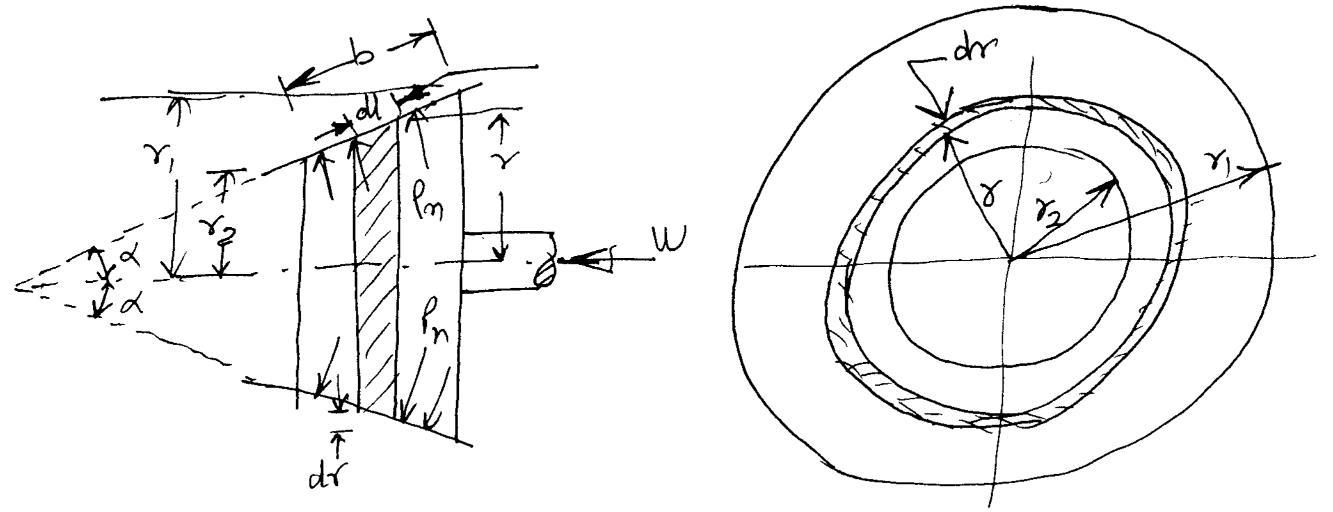
where, R = Mean radius of friction surfaces

$$= \frac{2}{3}\left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right] \quad \text{for Uniform pressure}$$

$$= \frac{r_1 + r_2}{2} \quad \text{for uniform wear}$$

Numerical: 10.22 to 10.30. of Theory of m/c by khurmi

Cone clutch:—



Consider a pair of friction surfaces as shown in fig. Since the area of contact of a pair of friction surfaces is a frustum of a cone.

Let p_n = Intensity of pressure with which the conical friction surfaces are held together (normal pressure betⁿ contact surfaces)

r_1 and r_2 = Outer and inner radius of friction surfaces respectively

R = mean radius of friction surface $= \frac{r_1 + r_2}{2}$

α = Semi angle of the cone

μ = Coefficient of friction between contact surfaces.

b = width of the contact surfaces

Consider a small ring of radius r and thickness dr, Let dl is length of ring of the friction surface, such that

$$dl = dr.\, \text{cosec}\,\alpha$$

Area of the ring,

$$A = 2\pi r\, dl = 2\pi r.\, dr.\, \text{cosec}\,\alpha$$

1. Considering Uniform pressure,

Normal loading on the ring,

$$\delta W_n = \text{Normal pressure} \times \text{Area of ring}$$
$$= p_n \times 2\pi r\, dr\, \text{cosec}\,\alpha$$

axial load acting on the ring,

$$\delta W = \text{Horizontal Component of } \delta W_n$$
$$= \delta W_n \times \sin\alpha = p_n \times 2\pi r\, dr\, \text{cosec}\,\alpha \times \sin\alpha$$

∴ Total Axial force Transmitted to the Clutch or the axial Spring force required.

$$W = \int_{r_2}^{r_1} 2\pi P_n\, r\, dr = 2\pi P_n \left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi P_n \left[\frac{r_1^2 - r_2^2}{2}\right]$$

$$= \pi P_n \left[r_1^2 - r_2^2\right]$$

$$P_n = \frac{W}{\pi\left[r_1^2 - r_2^2\right]}$$

friction force on the ring acting tangentially at radius r,

$$F_r = \mu\, \delta W_n = \mu \times P_n \times 2\pi r . dr . \operatorname{cosec}\alpha$$

Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu\, P_n \times 2\pi r\, dr\, \operatorname{cosec}\alpha\, r$$

$$T_r = 2\pi \mu P_n \operatorname{cosec}\alpha\, r^2\, dr$$

Total frictional torque,

$$T = \int_{r_2}^{r_1} 2\pi \mu P_n \operatorname{cosec}\alpha\, r^2\, dr$$

$$= 2\pi\mu P_n \operatorname{cosec}\alpha \left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$= 2\pi\mu P_n \operatorname{cosec}\alpha \left(\frac{r_1^3 - r_2^3}{3}\right)$$

Substituting the value of P_n from above Eq^n.

$$T = 2\pi\mu \times \frac{W}{\pi\left[r_1^2 - r_2^2\right]} \times \operatorname{cosec}\alpha \left[\frac{r_1^3 - r_2^3}{3}\right]$$

$$\boxed{T = \frac{2}{3}\mu W \operatorname{cosec}\alpha \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]}$$

2. Considering Uniform wear :—

In case of Uniform wear, Intensity of pressure varies inversely with the distance.

$$\therefore \; p_r \times r = C$$

$$\text{or} \quad p_r = \frac{C}{r}$$

where p_r normal intensity of pressure at a distance r.

Normal load acting on the ring

$$\delta W_n = p_r \times \text{Area of ring}$$

$$= p_r \times 2\pi r \, dr \operatorname{cosec}\alpha$$

Axial load acting on the ring,

$$\delta W = \delta W_n \times \sin\alpha = p_r \, 2\pi r \, dr \operatorname{cosec}\alpha \cdot \sin\alpha$$

$$\delta W = p_r \, 2\pi r \, dr$$

$$= \frac{C}{r} \, 2\pi r \, dr$$

$$\delta W = 2\pi C \, dr$$

Total Axial load transmitted to the clutch,

$$W = \int_{r_2}^{r_1} 2\pi C \, dr = 2\pi C \left[r \right]_{r_2}^{r_1}$$

$$W = 2\pi C (r_1 - r_2)$$

$$\text{or} \quad C = \frac{W}{2\pi (r_1 - r_2)}$$

Frictional force acting on the ring,

$$F_r = \mu \, \delta W_n = \mu \, p_r \, 2\pi r \, dr \operatorname{cosec}\alpha$$

Frictional Torque acting on the ring,

$$T_r = F_r \times r = \mu \, p_r \, 2\pi r \, dr \operatorname{cosec}\alpha \times r$$

$$= \mu \frac{C}{r} \, 2\pi r^2 \, dr \operatorname{cosec}\alpha$$

$$= 2\pi\mu \, C \operatorname{cosec}\alpha \times r \times dr$$

Total frictional Torque acting on the clutch,

$$T = \int_{r_2}^{r_1} 2\pi\mu C \operatorname{cosec}\alpha \, r \, dr = 2\pi\mu C \operatorname{cosec}\alpha \left[\frac{r^2}{2} \right]_{r_2}^{r_1}$$

$$T = 2\pi \mu C \operatorname{cosec}\alpha \left[\frac{r_1^2 - r_2^2}{2}\right]$$

Substituting the value of C,

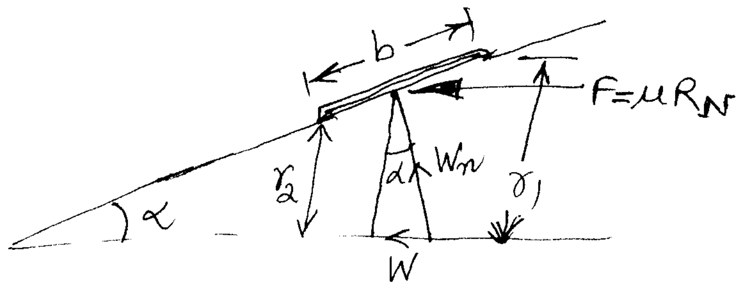
$$T = 2\pi\mu \times \frac{W}{2\pi(r_1 - r_2)} \times \operatorname{cosec}\alpha \left[\frac{r_1^2 - r_2^2}{2}\right]$$

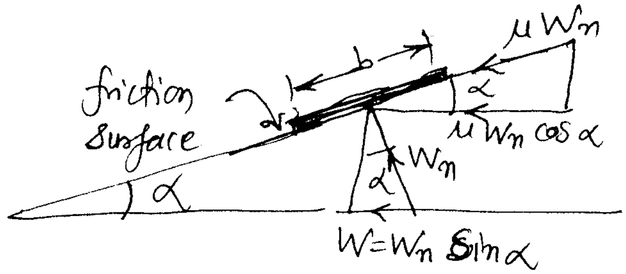
$$= \mu W \operatorname{cosec}\alpha \left(\frac{r_1 + r_2}{2}\right) = \mu W R \operatorname{cosec}\alpha$$

where, $R = \frac{r_1 + r_2}{2}$ = mean radius of friction surface.

$$W_n = \frac{W}{\sin\alpha}$$

$$\therefore \boxed{T = \mu W_n R}$$





$r_1 - r_2 = b \sin\alpha$ and $R = \frac{r_1 + r_2}{2}$ or $r_1 + r_2 = 2R$

$$P_n = \frac{W}{\pi[r_1^2 - r_2^2]} = \frac{W}{\pi(r_1+r_2)(r_1-r_2)} = \frac{W}{2\pi R b \sin\alpha}$$

or $W = P_n \times 2\pi R b \sin\alpha = W_n \sin\alpha$

where W_n = Normal load acting on the friction surface.

$$W_n = P_n \times 2\pi R b.$$

$$T = \mu(P_n \times 2\pi R b \sin\alpha) R \operatorname{cosec}\alpha$$

Single Block or Shoe Brake :-

A single block or shoe brake is shown in fig. It consists of a block or shoe which is pressed against the rim of a revolving brake wheel drum. Block is made of a softer material than the rim of the wheel.

The friction between the block & the wheel causes a tangential braking force to act on the wheel, which retard the rotation of the wheel. The block is pressed against the wheel by a force applied to one end of a lever to which the block is rigidly fixed. The other end of the lever is pivoted on a fixed fulcrum O.

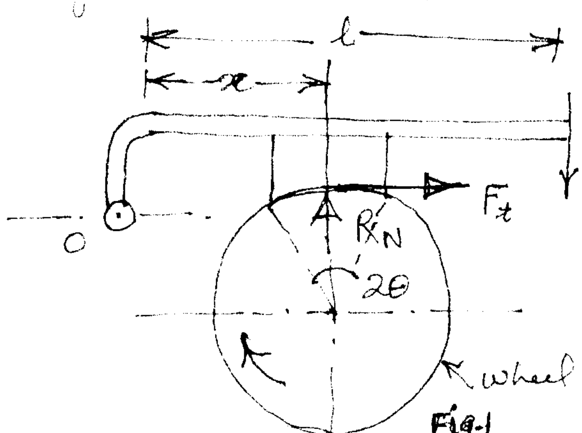


Fig-1

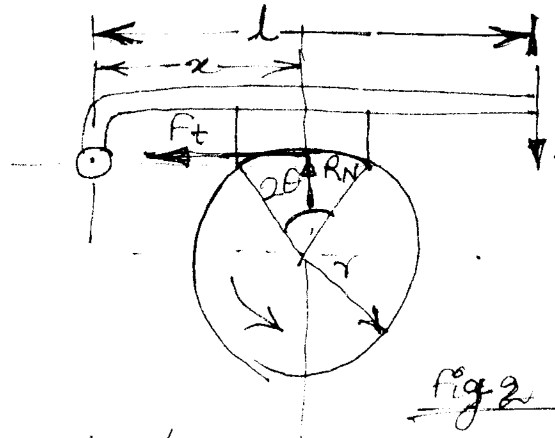


fig 2

Let P = Force applied at the end of the lever.

R_N = Normal force pressing the brake block on the wheel.

r = radius of the wheel

2θ = Angle of Contact Surface of the block

μ = Coefficient of Friction.

F_t = Tangential braking force or the frictional force

Tangential Braking Force,

$F_t = \mu R_N$

and the Braking Torque, $T_B = F_t \cdot r = \mu R_N \cdot r$

Case 1 :- from **Fig 1**:

$$R_N \times x = P \times l \quad \text{or} \quad R_N = \frac{P \times l}{x}$$

Braking Torque,

$$T_B = \mu R_N \cdot r = \mu \times \frac{P \times l}{x} \times r = \frac{\mu P l r}{x}$$

$$\boxed{T_B = \frac{\mu P l r}{x}}$$

Brakes:— A brake is a device by means of which artificial frictional resistance is applied to a moving machine member, in order to retard or stop the motion of a machine, & the brake absorbs either kinetic energy of the moving member or potential energy given up by objects. The energy absorbed by brakes is dissipated in the form of heat. This heat is dissipated in the surrounding air or water so that excessive heating of the brake lining does not take place.

The capacity of brake depends upon the following factors:

1. The unit pressure between the braking surfaces.
2. The coefficient of friction between the braking surfaces
3. The peripheral velocity of the brake drum.
4. The projected area of the friction surfaces and
5. The ability of the brake to dissipate heat equivalent to the energy being absorbed.

Material for Brake Lining:

The material used for the brake lining should have the following characteristics:

1. It should have high coefficient of friction with minimum fading.
2. It should have low wear rate.
3. It should have high heat resistance
4. It should have high heat dissipation capacity.
5. It should have adequate mechanical strength.
6. It should not be affected by moisture and oil.

Types of Brakes:

1. Hydraulic Brakes
2. Electric ~~or~~ Brakes.
3. Mechanical Brakes.

Mechanical Brakes, according to the direction of acting force, may be divided into the following two groups.

1. Radial Brakes.
2. Axial Brake.

Radial Brake may be subdivide into 'External brakes' and 'Internal Brakes'. According to the shape of the friction elements.

Case-2 :- When the line of action of the tangential braking force (F_t) passes through a distance 'a' below the fulcrum O, and the brake wheel rotates clockwise as shown in Fig. 1

taking Moment about O,

$$R_N \times x + F_t \times a = P \times l \quad \text{or} \quad R_N x + \mu R_N a = P \times l$$

$$\text{or} \quad R_N = \frac{P \times l}{x + \mu a}$$

and braking torque, $T_B = \mu R_N \cdot r = \dfrac{\mu \times P \times l \times r}{x + \mu a}$

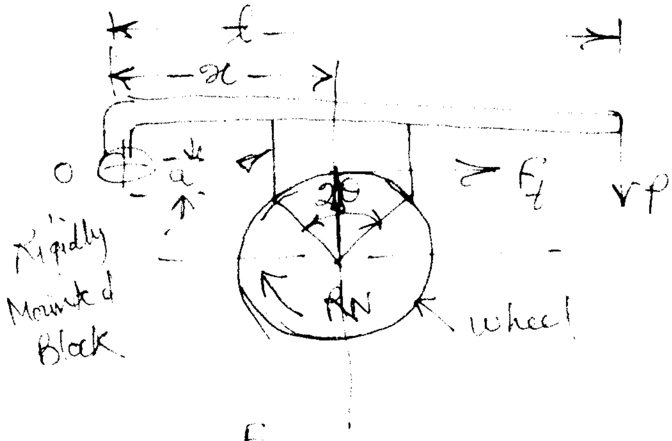


Fig 1.

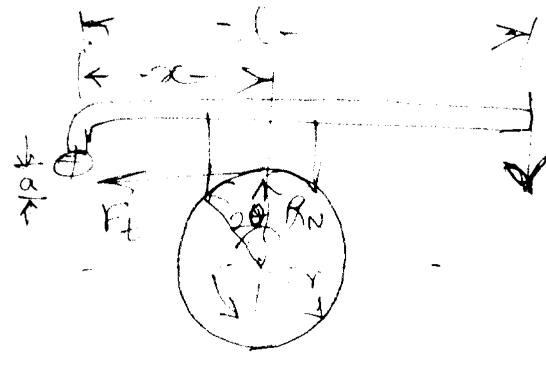


Fig 2

When the brake wheel rotates anticlockwise,

$$R_N \times x = P \times l + F_t \times a = P \cdot l + \mu R_N a$$

$$R_N (x - \mu a) = P \cdot l$$

$$\text{or} \quad R_N = \frac{P \cdot l}{(x - \mu a)}$$

and Braking Torque, $T_B = \mu R_N \cdot r = \dfrac{\mu P l r}{(x - \mu a)}$

Case 3:– When the line of action of the tangential force (F_t) passes through a distance 'a' above the fulcrum 'O'.

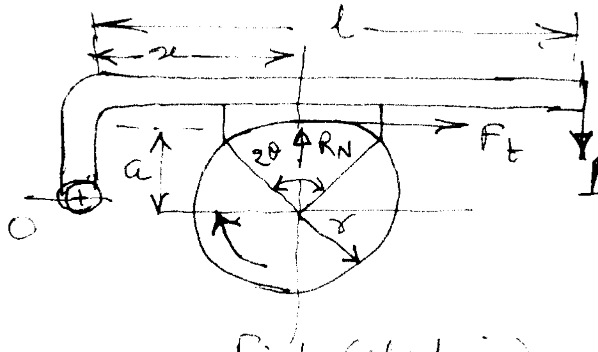


Fig 1 (clockwise)

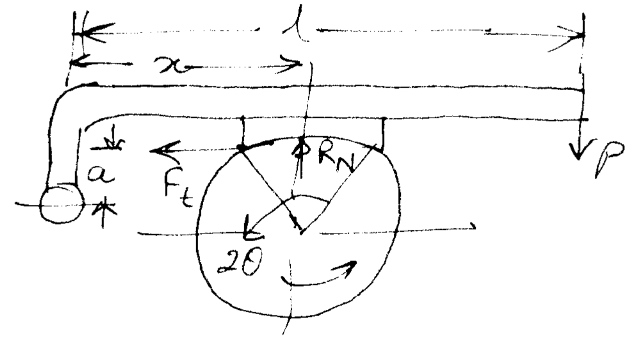


Fig 2 (Anticlockwise)

Brake wheel rotates clockwise, then taking moment about the fulcrum O, we have.

$$R_N x = P l + F_t \cdot a = P \cdot l + \mu R_N a$$

$$R_N(x - \mu a) = P \cdot l \quad \text{or} \quad R_N = \frac{P \cdot l}{(x - \mu a)}$$

and braking Torque, $T_B = \mu R_N \cdot r$

$$\boxed{T_B = \frac{\mu P l r}{(x - \mu a)}}$$ clockwise

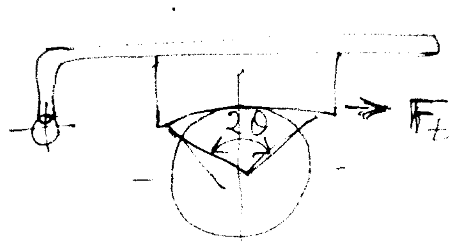
when the brake rotate anticlockwise, taking Moment about the fulcrum O,

$$R_N \times x + F_t \times a = P \cdot l \quad \text{or} \quad R_N \times x + \mu R_N \times a = P \times l \quad \text{or}$$

$$R_N = \frac{P l}{(x + \mu a)}$$

and Braking Torque, $T_B = \mu R_N \cdot r = \boxed{\frac{\mu P l r}{(x + \mu a)}}$

Note: when the angle of contact is greater than $60°$.



$T_B' = F_t \times r$

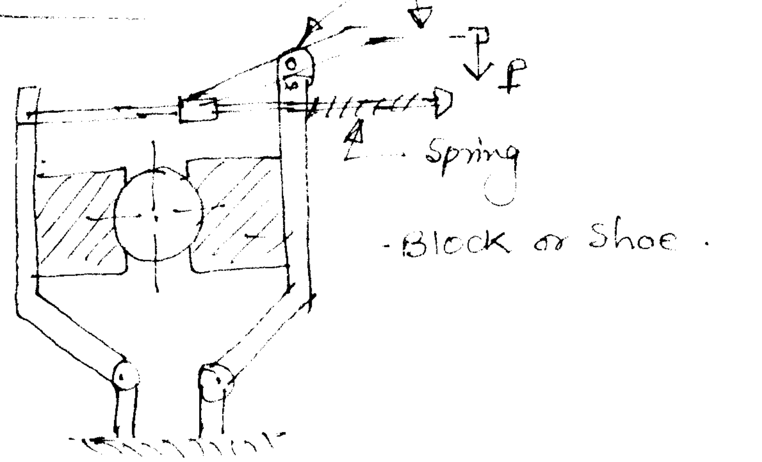
$2\theta > 60°$

$T_B' = \mu' R_N r$

$$\mu' = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta}$$

Ex- Ref 19.1 - to 19.4.

Double Block or Shoe Brake :—



In a double brake,

Braking Torque,

$T_B' = (F_{t_1} + F_{t_2}) r$

Example 19.5

Simple Band Brake :— A band brake consists of a flexible band of leather, one or more ropes, or a steel lined with friction material, which embraces a part of the circumference of the drum. ~~A~~ brake called a "Simple Band brake" in which one end of the band is attached to a fixed pin or fulcrum of the lever while the other end is attached to the lever at a distance b from the fulcrum.

When a force P is applied to the lever at C, the lever turns about the fulcrum pin O and tightens the band on the drum and hence the brakes are applied. The friction between the band and the drum provides the braking force. The force P on the lever at C may be determined as discussed below :–

Let T_1 = Tension in the tight side of the band,

T_2 = Tension in the slack side of the band,

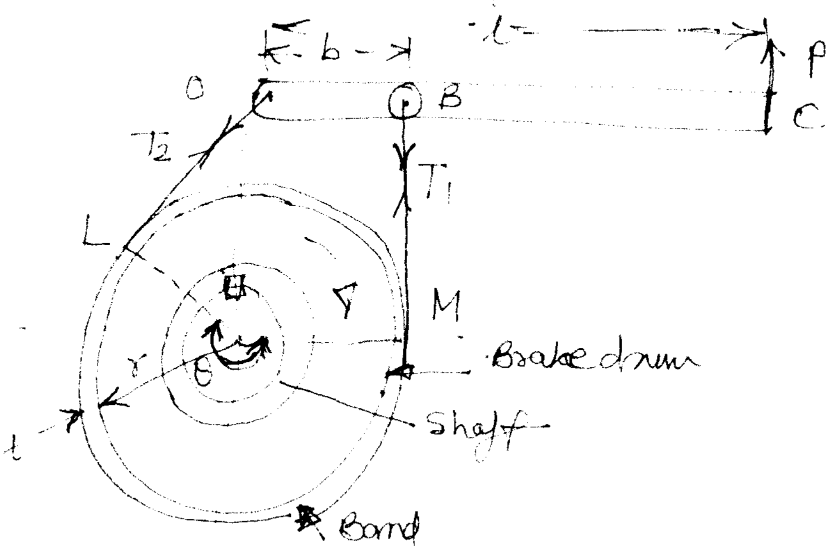
θ = angle of lap of the band on the drum,

μ = Coefficient of friction between the band and the drum

r = radius of the drum.

t = thickness of the band

r_e = Effective radius of the drum $= r + \frac{t}{2}$



Limiting ratio of the tension is given by the relation,

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

[illegible]

[illegible] attached to the fulcrum O will be slack with Tension T_2
and end of the band attached to B will be tight with Tension T_1,

So Moment about O,

$P \cdot l = T_1 \cdot b$ for clockwise

[illegible] for [illegible]

Example 13.6 to 13.8